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Optimizing Spatial Learning with RME (Realistic Mathematics Education) to Sharpen Conceptual Understanding

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Abstract

This study aims to design and test the effectiveness of interactive didactic designs for teaching Flat Surface Shapes to eighth graders. This didactic design specifically integrates the Realistic Mathematics Education (RME) approach with game technology as the primary supporting medium. The methodology used is Didactical Design Research (DDR), which involves three cycle phases: 1) Preliminary Research, to analyze needs and identify three types of Learning Obstacles—namely ontogenetic, epistemological, and didactic—faced by students; 2) Testing Phase (Prototyping/Testing) through pilot experiments and teaching experiments; and 3) Retrospective Analysis to evaluate the learning process. The research participants were eighth-grade students. Research Results: The resulting didactic design proved to be highly valid theoretically (with an average score of 3.86 on a scale of 4) and highly practical in implementation, with 93.1% of students responding positively to the use of games. The analysis shows that this DD reduced the number of students who experienced limitations in conceptual understanding across test cycles. The integration of RME and games proved effective because games serve as a dynamic modelling medium, helping students transition from a contextual understanding to the formalization of mathematical concepts (for example, in calculating volume). This DD produced an empirically supported Local Learning Trajectory (LLT), indicating that the effectiveness of spatial learning is enhanced when the RME process is supported by game media that facilitates the transition from situational to formal thinking.

Keywords: *Didactic Design; Flat-Sided Spatial Figure; Interactive Games; Local Learning Trajectory (LLT); Realistic Mathematics Education (RME)*

Introduction

Geometry is very useful in life. Geometry is a crucial tool for communication and for applying to real-life problems and topics in basic mathematics (Hwang et al., 2019). Geometry facilitates students to develop spatial abilities and represent them in other forms. For example, in understanding the properties of a flat shape, students observe the shape. One of the geometry lessons covers spatial shapes. The topic of flat-sided spatial shapes can reveal size, number, shape, and space.

Mathematics learning, especially spatial geometry, can be challenging for eighth-grade junior high school students. This is due to the high demands on students' spatial visualization abilities (Mandala, Anwar, Sad'dijah, & Zulnadi, 2025). Three-dimensional and abstract spatial geometry concepts demand advanced visualization and spatial skills (Jones, 2020). Unfortunately, conventional learning processes dominated by lectures and two-dimensional models (such as pictures in books) often fail to bridge students' understanding from concrete forms to formal mathematical concepts (Smith & Brown, 2019). As a result, many students find this material difficult, irrelevant to their daily lives, and demotivating (Kemendikbud, 2021). Geometry learning is entirely deductive-axiomatic (Zhang & Wong, 2021). Proof and reasoning play an important role in geometry learning. However, students often struggle to prove geometric principles, particularly during the process of mathematizing spatial shapes.

Spatial learning encompasses basic concepts of flat shapes and cube and block shapes, which are typically introduced in elementary school. Misunderstanding or lack of understanding of these concepts creates learning obstacles that can hinder further learning in mathematics. The mathematics learning process is influenced by students' perspectives on mathematics itself (Brown, 2016). Students often experience difficulties in learning mathematics (Yasar, 2016), particularly with abstract and challenging topics such as solid geometry. Students also experience misconceptions in learning various mathematical topics (Abreu-Mendoza et al., 2023; Bush & Karp, 2013). Some learning obstacles when dealing with solid geometry topics include the use of incorrect formulas and a misunderstanding of concepts (Özerem, 2012). Supported by Sudirman's findings, several obstacles in learning solid geometry have been identified, including students' inability to visualize unit cubes that are not visible at each level, such as determining the structure of the edges of a unit cube located on the base (Sudirman et al., 2023). Therefore, it is essential to instill a proper understanding of concepts to help students overcome their learning obstacles. Inappropriate strategies need to be implemented in the learning process.

Alternative learning methods are needed to meet students' needs in mathematics. One way is to design a learning trajectory for spatial geometry material. Simon (1995) presents the Hypothetical Learning Trajectory (HLT) as a means to explain key aspects of pedagogical thinking involved in teaching mathematics for student understanding (Simon, 2004). HLT is designed to offer an empirically grounded model of pedagogical thinking, grounded in constructivist ideas (Simon et al., 2018). With HLT, mathematical logic is rediscovered based on mathematical activities (Hub & Dawkins, 2018). Learning trajectories in geometry learning refer to the steps students take in their learning activities to understand geometry. The Hypothetical Learning Trajectory (HLT) predicts how students' thinking and understanding will evolve during the lesson and how their understanding will develop through a series of educational activities, with learning objectives that must be met in a single application (Hendriana et al., 2019). A paradigm shift is needed from teacher-centred learning to a more student-centred and contextual approach. One approach that has proven effective is Realistic Mathematics Education (RME). RME offers a framework in which mathematical concepts are constructed through reinvention in real contexts that are familiar to students (Treffers, 1987; Freudenthal, 1991). In the context of spatial figures, RME enables students to understand the concepts of volume and surface area by relating them to practical situations, such as designing packaging or calculating the quantities of building materials required for a project.

However, in order for RME contextual learning to be more effective and appealing to the digital generation, the integration of technology is crucial. In this context, gaming technology serves not only as a means of entertainment but also as a powerful interactive learning medium (Prensky, 2001). Games are capable of providing real-time 3D visualization, facilitating the virtual manipulation of spatial objects, and providing instant feedback (Veen & Van de Laar, 2022). This directly supports the development of spatial abilities while increasing their motivation and engagement in the learning process (Gee, 2003; Prensky, 2001). The combination of games and RME has excellent potential to "bring to life" previously rigid and abstract spatial concepts into a dynamic and immersive learning experience.

Based on the above background, this article aims to explain and describe in detail the didactic design of interactive spatial learning for eighth-grade students. This design explicitly integrates the steps of Realistic Mathematics Education, utilizing game technology as the primary supporting medium. This design presentation will serve as a practical reference for teachers and curriculum developers in designing a more effective and meaningful learning experience in solid geometry, thereby increasing student motivation and conceptual understanding.

Method

This study employs Didactical Design Research (DDR), also known as Design Research in the context of mathematics education, as its primary methodological approach (Plomp, 2013; Gravemeijer & Cobb, 2006). Design research is a type of research that builds theories about learning processes, such as activities or learning paths for specific topics. This design research was used to create a learning design called mathematical didactic design, which was packaged with the RME approach (Plomp, 2013). The purpose of DDR is not only to produce a didactic design product but also to develop a theory (Design Hypothesis) about how the spatial learning process can be effectively integrated through the combination of RME and games. The resulting didactic design serves as an artefact that is tested and refined through repeated cycles, ensuring both theoretical and practical contributions. This research was conducted through three phases of the DDR cycle, namely the preliminary research stage, the prototyping/testing phase, and the retrospective analysis stage.

The initial phase involved analyzing the didactic situation before implementing HLT in the classroom. At this stage, the researcher conducted a prospective analysis to create a didactic framework for teaching the material on surface area and volume of flat-sided shapes, taking into account the learning obstacles faced by students. The researcher analyzed the needs based on teacher interviews, curriculum reviews, literature reviews, and tests administered to students to identify learning obstacles. Based on this analysis, an initial didactic design was formulated that predicted the learning flow of students from real-world contexts (RME) to the formalization of solid geometry concepts, aided by game visualization.

Next, during the testing phase, it will be tested in two classes: the pilot experiment class and the teaching experiment class. The initial didactic design was tested in a limited trial class (pilot experiment). At this stage, in-depth observation data were collected to examine how students interacted with the game and how the RME context functioned (Gravemeijer & Cobb, 2006). This data is then used to adjust the initial design for data collection in the teaching experiment class. The teaching experiment aims to study and refine the Learning Pathway Hypothesis (LPH) and gain a deep theoretical understanding of how students learn.

The final phase is Retrospective Analysis, in which data from the teaching experiment class will be carefully analyzed. This analysis aims to evaluate whether the observed learning process aligns with the formulated design. The results of this analysis will form the basis for improving and producing HLT that is supported by empirical theory.

Results and Discussion

This study identifies three main types of obstacles in mathematics learning, namely ontogenetic obstacles, which include students' difficulties in contextual story problems, lack of understanding of spatial structures and pyramid volumes, and a tendency to memorize formulas; epistemological obstacles characterized by misconceptions and limited understanding of the concepts of nets and the volume of flat-sided shapes; and didactic obstacles stemming from teaching practices that focus solely on textbooks, minimal use of media by teachers, and a direct transition to formal mathematics without facilitating the discovery of concepts by students.

Based on the learning obstacles above, several didactic situations were compiled into an initial didactic design. Students had the opportunity to reflect on their learning in HLT, which was designed to help them better understand the concepts of surface area and volume of flat-sided shapes. The use of games would help students recall the concepts of surface area and volume of flat shapes more enjoyably. The following is an image of the initial design, compiled based on the prospective analysis conducted.

Figure 1. HLT surface area of flat-sided solid figures

Figure 2. HLT volume of flat-sided solid figures

The design is outlined in the Student Worksheet, which consists of four worksheets. Student Worksheet 1 aims to teach students about types of spatial structures, their characteristics, and spatial structure grids. Worksheet 2 aims to teach students about the surface area of cubes and blocks. Worksheet 3 aims to help students understand the surface area and volume of pyramids. Worksheet 4 aims to help students understand the surface area and volume of prisms. Contextual examples accompany each worksheet, and a game is included at the end of the lesson.

Based on testing in a limited pilot experiment given to 24 students, there were three types of obstacles in the pretest, process, and posttest. Ontogenetic obstacles in students were evident from the pretest, with a tendency to guess the shape of the nets, calculate only one side for the surface area of the cube, and memorize the volume without understanding the underlying concept. Although students showed responses in line with HLT predictions during the process stage, they still required guidance and often made calculation errors or wrote down the incorrect area/volume units. In the posttest, students' answers were correct but often incomplete, still containing errors in calculation and the use of units. Epistemological obstacles were identified from students' initial difficulties in drawing nets, misunderstanding the order of sides, difficulty determining the height of triangles in pyramids, and misunderstanding the concept of volume (always assuming the multiplication of three elements). During the learning process, these difficulties persisted in distinguishing between the abstract concepts of space diagonals and plane diagonals, as well as identifying the length and width of nets of blocks with different sizes. On the posttest, some students felt confused when drawing nets, even though they already knew the shape of the flat figures, although most students were able to draw them.

Figure 3. Student responses to the pilot experiment cycle for spatial construction networks

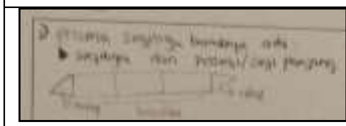
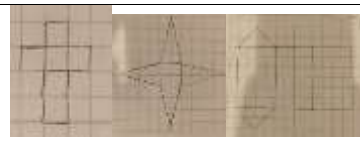
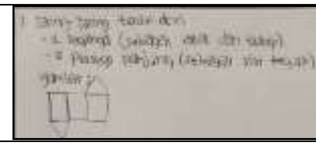
Pretest	Process	Posttest
		

Figure 4. Student responses to the pilot experiment cycle for surface area

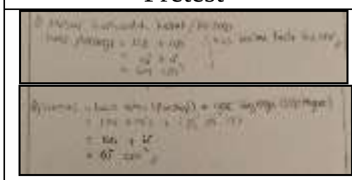
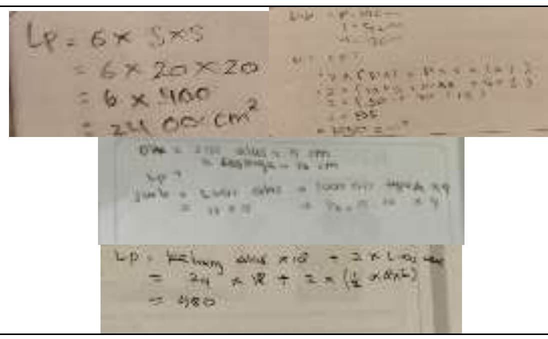
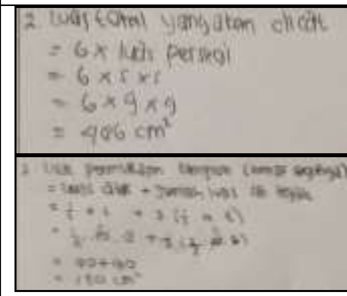
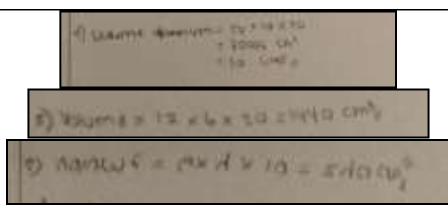
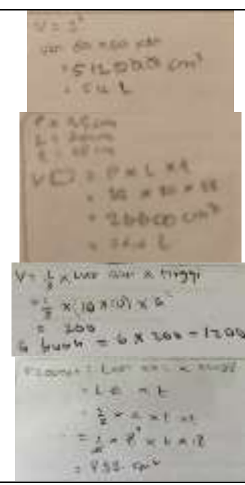
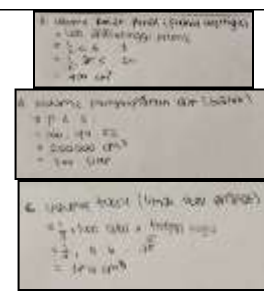
Pretest	Process	Posttest
		

Figure 5. Student responses to the pilot experiment cycle for spatial volume

Pretest	Process	Posttest
		

The revisions made included adding guidance to Worksheet 1 on sorting gifts, modifying the instructions in Worksheet 3 for calculating the surface area of a pyramid, and updating the instructions in Worksheet 4 for calculating the surface area of a prism. There were 28 students involved as research participants in this teaching experiment cycle. The ontogenetic barriers (students) identified in this class included the incompleteness and accuracy of student work (calculation/unit errors, incomplete answers, memorization without understanding the concept, and a lack of thorough understanding of the questions). Meanwhile, students' epistemological obstacles were fundamental misconceptions about abstract spatial (networks) and quantitative (volume) concepts, such as thinking that a prism is a pyramid or only adding up some of the vertical sides. The number of students who experienced limitations in understanding the material on surface area and volume of spatial figures was also smaller compared to the pilot experiment cycle. This shows that the designed didactic design was able to reduce the learning obstacles faced by students.

Figure 6. Student responses to the teaching experiment cycle for spatial construction networks

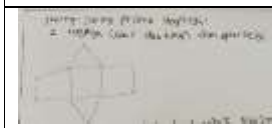
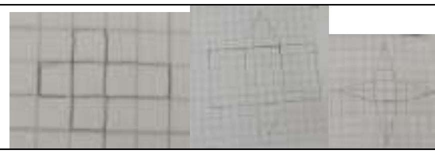
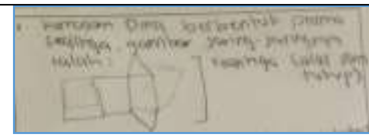
Pretest	Proses	Posttest
		

Figure 7. Students' answers in the teaching experiment cycle for surface area

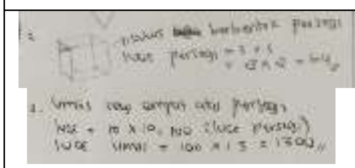
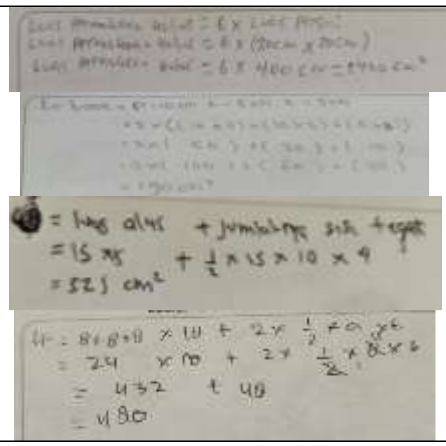
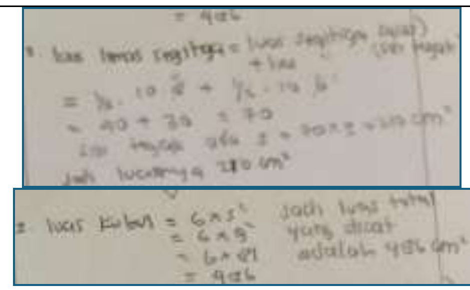
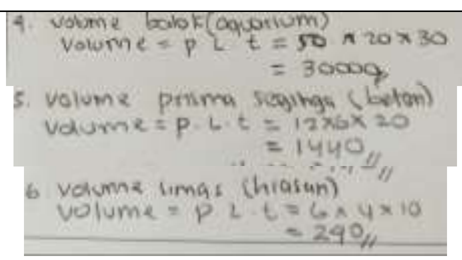
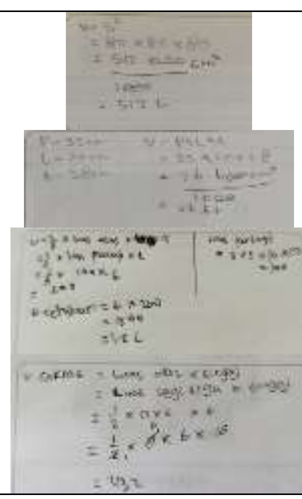
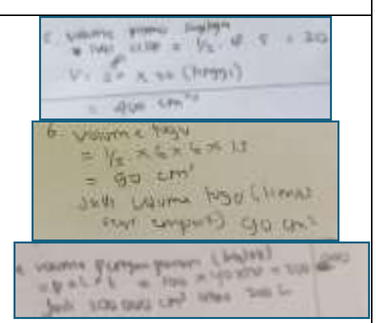
Pretest	Proses	Posttest
		

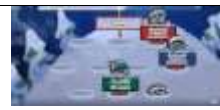
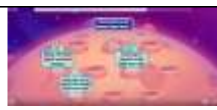
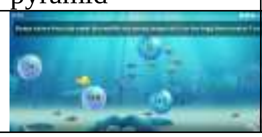
Figure 8. Student responses to the teaching experiment cycle for the volume of solid figures

Pretest	Proses	Posttest
		

In the didactic design developed in this study, there is a greater possibility that students will find the learning atmosphere more enjoyable. After the lesson, the game functions as a quiz. The game design aims to help students understand what they have learned. Students can access the game by scanning a barcode with their cell phones. Each group can complete each question in the game, based on the findings of the pilot experiment and teaching experiment cycles. However, not all questions can be answered correctly. As a result of brief interviews conducted with students, they reported enjoying the game, even though some questions required more complex calculations.

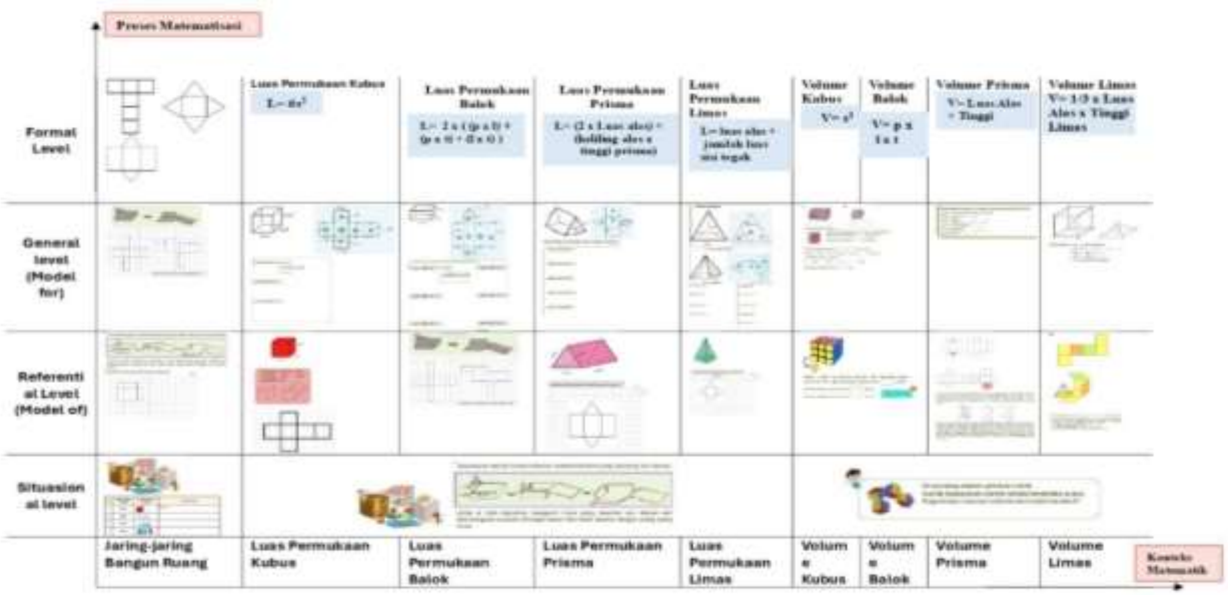
Additionally, the reason students in certain groups accidentally chose the wrong answers was that students in other groups were confused by the other choices. The above description shows that games can help students improve their problem-solving and collaboration skills. It also demonstrates that games can enhance students' understanding of the concepts that have been applied. Examples of games in student activity worksheets are presented in the following image.

Figure 9. Game

classification of spatial structures	characteristics of spatial structures	characteristics of a cube	characteristics of blocks	surface area of a cube and a rectangular prism
				
volume of cubes and blocks	characteristics of a pyramid	surface area and volume of a pyramid	characteristics of a prism	surface area and volume of a prism
				

This study demonstrates that students' understanding of the surface area and volume of flat-sided shapes can be enhanced through the use of terminology and student interaction when employing the RME approach. The local learning trajectory for learning the surface area and volume of flat-sided shapes is illustrated in the figure below.

Figure 10. LLT Learning Surface Area and Volume of Flat-Sided Shapes



Conclusion

This study aims to design and test the effectiveness of interactive didactic designs for teaching flat-sided shapes to eighth graders, integrating the Realistic Mathematics Education (RME) approach and the use of game technology. Learning Obstacles: Three main types of obstacles were identified in students, namely ontogenetic, epistemological, and didactic. Ontogenetic obstacles include students' difficulties with contextual story problems, a lack of understanding of flat-sided shapes/pyramid volume, and a tendency to memorize formulas. Epistemological obstacles arise due to misconceptions about abstract nets, the volume of flat-sided shapes, and a limited understanding of related concepts. Meanwhile, didactic obstacles occur in conventional learning practices that focus on textbooks, the minimal use of media/teaching aids by teachers, and the direct transition to formal mathematics without facilitating concept discovery. The didactic design developed was able to reduce the learning obstacles faced by students, as evidenced by the decrease in the number of students who experienced limited understanding in the teaching experiment cycle compared to the pilot experiment. The RME approach combined with games has been proven to improve students' understanding of the surface area and volume of flat-sided shapes. The use of games at the end of the lesson serves as a fun review, helping students reinforce concepts and enhance their problem-solving and collaboration skills. This design produces a Local Learning Trajectory (LLT) supported by empirical theory, which predicts the learning path of students from a real-world mathematics education (RME) context to the formalization of solid geometry concepts, aided by games. Thus, the integration of RME and game technology in the didactic design of solid geometry learning provides an effective practical reference for designing more meaningful and interactive learning experiences that can overcome the conceptual difficulties experienced by students. The Local Learning Trajectory (LLT) theory provides empirical guidance that the effectiveness of spatial learning is enhanced when the RME process is supported by game media that facilitates the transition from situational thinking to formal thinking. This design offers an innovative solution for creating a more engaging, meaningful, and contextually problem-solving-oriented mathematics learning environment.

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