



Mathematical Abstraction of Junior High School Students on Function Based on Gender Perspectives

Nor Khasanah; Tri Atmojo kusmayadi; Farida Nurhasanah

Sebelas Maret University, Indonesia

<http://dx.doi.org/10.18415/ijmmu.v8i5.2602>

Abstract

The purpose of this study is to describe the mathematical abstraction of junior high school students in learning the concept of function between male and female students. This is a qualitative descriptive study with total participants were 28 students in grade 8th. There were 14 female students and 14 male students in two different classes. The data were collected through written tests and interviews. In this study, the researcher gave abstraction test questions. Furthermore, from the test results, the interview was conducted. The researcher chose 4 communicative students from the high ability. Based on the data analysis result, mathematical abstraction of the students can be classified into 4 levels: 1) Recognition, 2) Representation, 3) Structural Abstraction and 4) Structural Awareness. The result of research showed that female students achieved mathematical abstraction levels, namely: recognition, representation, structural abstraction and structural awareness. Meanwhile, male students only achieved the level of recognition, it means that commonly the male students do not yet have the mathematical abstraction ability in function concepts. Therefore, it is necessary to conduct further research on the differences in the abstraction process of male and female students in learning context using the SCL approach.

Keywords: *Mathematical Abstraction; Algebra; Gender Perspectives*

Introduction

Mathematics is one of the important subjects that must be taken by the students. Mathematics is a subject that deals with many concepts. The concepts in mathematics are abstract. The students are difficult in learning mathematics if the students emphasize memorization. Therefore, the students should be given more opportunity to build and construct their own knowledge through meaningful learning or mathematical concepts should not be transferred by the teacher directly to the students.

The concept construction process that occurs in the students' mind by utilizing their early experience or knowledge is called the process of mathematical abstraction (Nurhasanah, Kusumah, & Sabandar, 2017). The abstraction process will exist from someone's experience, namely the students' experience in building the early knowledge and concepts (Fitriani, Suryadi, & Darhim, 2018). So, mathematical abstraction is a process that is related to the emergence of mathematics concepts, which means that this process is crucial in mathematics learning (Nurhasanah, Sabandar, & Kusumah, 2013).

The main abstraction is looking for the same or common from a community in real examples (Hershkowitz, Schwarz, & Dreyfus, 2001). Looking for the same characteristics is a basic classification. The similarity used as basic qualification, and then people can know the new experience by comparing the existing experience in previous thoughts (Fitriani, Suryadi, & Darhim, 2018). The new mathematical experience that has been built in previous is called abstraction. The result of the abstraction process is a concept. Those statements are in line with a Posteriori theory which states that all knowledge begins with experience (Kant, 1855).

According to the definition above, concept formation in the abstraction process will occur every time students learn about something new and will materialize based on students' previous experiences. So, the abstraction process is crucial to be mastered by the students in learning mathematics, especially in the formation of mathematical concepts.

With regard to abstraction, Piaget (1972) and Piaget (2001) introduced three kinds of abstraction, namely empirical abstraction, pseudo-empirical abstraction, and reflective abstraction. Meanwhile, Mitchelmore & White (2007) divide abstraction into two stages, namely empirical abstraction and theoretical abstraction. In Mitchelmore and White's conception the abstraction consists of the formation of concepts that are in accordance with several theories. This means that new concepts will emerge when students can match existing concepts in previous experiences. Therefore, it is called a theoretical abstraction. This view is related to Tall (1991) statement who claims Piaget's theory of reflective abstraction is a form of theoretical abstraction. Meanwhile, reflective abstraction consists of four levels, namely: recognition, representation, structural abstraction, and structural awareness (Cifarelli, 1988).

Some of the experts who have studied about the abstraction as follows: (Subroto & Suryadi, 2018); (Putra, Suryadi, & Juandi, 2018); (Muniroh, Usodo, & Subanti, 2017); (Komala, 2018); etc. The result of study commonly shows that still weak in abstraction ability of mathematics learning, especially in algebra. Algebra is one of the topics which can be used to develop the abstraction of mathematical algebra. Algebra is also a core topic in mathematics taught since junior high school and for the implication can be found in various topics of mathematics such as: analytic geometry, calculus, statistics, trigonometry, vector, matrix, and topology (Jupri, Drijvers, & Heuvel-Panhuizen, 2014). Algebra is one of the objects of mathematical study which can be seen from the abstract. The abstract characteristics of mathematics is one of the reasons students have difficulty in learning algebra. The students said that the difficulties in understanding the characteristics of algebra such as symbols, to verify the characteristics of algebra, to connect one concept to another concept, and to solve the problems of algebra if there is no example from the teacher (Putra, Suryadi, & Juandi, 2018). In addition, when the students are asked to explain the result of their calculations, they are confused in presenting and abstracting (Komala, 2018)

The confusion is related to the way of forming abstract concepts. The concept formation must be done independently by the students, and cannot be done if just transfer the knowledge. Furthermore, the students are giving the wide opportunity to build and construct their own experiences that happened. So, it is crucial to assess the students' mathematical abstraction ability in algebra.

In learning mathematics, many factors that must be considered. One of the factors that give the influence to the students in mathematical abstraction ability is gender. Gender is the characteristic that different in every individual student in learning and processing information. Gender is seen as an analytic factor that humans use to think and organize their social activities (Harding, 1986). The gender differences are certainly caused by the physiological as well as affect psychology in learning, so that male and female students have differences in learning mathematics (Mawaddah, Ahmad, & Duskri, 2018). When it comes to the gender differences, it is certainly a contributing factor between the mathematical abstraction ability of both male and female students. The difference is seen in how male and female students solve the problems. Thus, the differences make a gap between male and female students.

Based on the previous explanation above, this study is to describe the mathematical abstraction ability of junior high school in relation and function. This study is giving information about the way to produce the mathematical abstraction ability of the students. This information is helpful for the researcher and mathematics teacher in designing the learning strategy for mathematical abstraction ability.

Methodology

In this study, researchers used a qualitative descriptive method. This research was conducted at a private junior high school in Tegalsambi, Jepara, and Central Java, Indonesia. The total number of participants was 28 students of class VIII, consisting of 14 female students and 14 male students in two different classes. Subjects were selected based on the results of the mathematical abstraction ability test. Based on the mathematical abstraction ability test data, the researcher chose four commutative students of high ability by purposive sampling. Researchers choose students with good communication skills because researchers want students who are able to provide or convey ideas and reasons. The selection of students is also based on the results of the consideration of the researcher and the mathematics teacher who teaches the class, this is because the teacher knows more about the characteristics of their students. Based on the results of the consideration of the researcher and class VIII mathematics teacher, the research subjects consisted of two male and two female students.

The instruments in this study were mathematical abstraction tests and interview guides. In collecting data, it was done by means of tests and interviews. Data were collected from the students' mathematical abstraction test by giving one question about the function, where the question represented the level of characteristics and activity of the student's mathematical abstraction. Based on these data, students' mathematical abstraction can be classified into four, namely recognition, representation, structural abstraction, and structural awareness. This finding is in line with Cifarelli (1988) theory. In this study, researchers used a modified standard of analysis based on the level of characteristics and reflective abstraction activity from Cifarelli (1988) and Fitriani, Suryadi, & Darhim (2018). Table 1 below is a table of the characteristics and activities of mathematical abstraction.

Table 1. Characteristic and Activity Level of Mathematical Abstraction.

| Abstraction Level in Reflective Abstraction | Characteristic and Activity |
|--|---|
| Recognition | Recall previous activities and experiences related to the problems at hand. |
| Representation | Be able to solve the problems by anticipating any source of difficulty (by first stating the thoughts result in the form of mathematical symbols, words or diagrams). |
| Structural Abstraction | Reorganise (collect, assemble, develop) mathematical elements into new elements. |
| Structural Awareness | Give reasons (formulas/rules) to the resulting decisions. Be able to show the summary of their activities. |

Data triangulation used to be carried out to decide the validity of the data. The data triangulation used was time triangulation. Data analysis in this research are data reducing, data presenting, verification, and conclusions drawing (Wulandari & Wutsqa, 2019). Data analysis procedures were:

The form of the mathematical abstraction test questions is as follows:

Problem:

"A linear function with a value of **2** when $x = -1$ and a value of **11** when $x = 2$. Find the formula for the function and the value of $f(3)$!"

Furthermore, the data is collected and analyzed descriptively. Data analysis techniques in this study include: 1) data reduction, 2) data presentation, and 3) conclusions.

Result and Discussion

The following is an analysis of data and the results of male and female students' work based on a mathematical abstraction test.

Recognition

Based on the result of the students' mathematical abstraction ability test for the recognition level, male and female students are able to recall previous activities and experiences related to the concept of function that has a value at time x and make predictions to solve the questions by applying the concept of the function f is linear function, it means that the recognition level is easy for students. This is in line with Fitriani, Suryadi, & Darhim (2018) who stated that all students had adequate recognition skills. Here is one of the examples of the student answer in recognition level the problem.

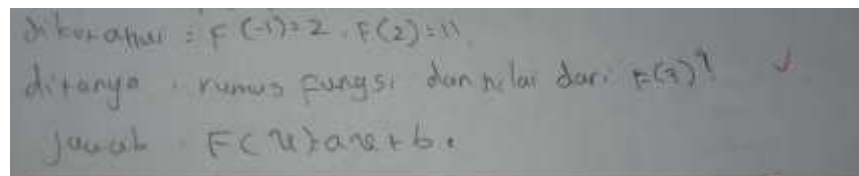


Figure 1. The Answer of a Student in Recognition Level

Representation

Based on the results of the students' mathematical abstraction ability test for the level of representation, it appears that female students can anticipate the difficulties faced by making the linear function $f(x)$ into an equation. Whereas for the two male students, they could not anticipate their difficulties and were mistaken in rearrange the elements, so the levels of representation and structural abstraction are passed (Cifarelli, 1988). Thus, structural awareness is only experienced by students with high ability (Fitriani, Suryadi, & Darhim, 2018). Here is one of the examples of the student answer in representation level the problem.

Figure 2. The Answer of a Student in Representation Level

Structural Abstraction

Based on the results of the students' mathematical abstraction ability test for the level of structural abstraction, it appears that female students are already at the third level, namely the level of structural abstraction. Meanwhile, the two male students have not reached this level, the error of the two male students occurs when they cannot rearrange the obtained equation into the form of elimination and substitution. Lack of practice in rearranging mathematical elements into new elements caused by the students' low mathematical abstraction ability. It shows that female students are more careful in solving problems than male students (Mawaddah, Ahmad, & Duskri, 2018). Here is one of the examples of the student answer in structural abstraction level the problem.

The image shows a student's handwritten solution for a system of linear equations. The work is divided into two parts: elimination and substitution.

Eliminasi pers. (1) dan (2)

$$\begin{array}{r} -a + b = 2 \\ a + b = 11 \\ \hline 2a = 9 \\ a = \frac{9}{2} = 4,5 \end{array}$$

Substitusi nilai a ke persamaan (2)

$$\begin{array}{r} a + b = 11 \\ 4,5 + b = 11 \\ b = 11 - 4,5 \\ b = 6,5 \\ a = 4,5 \quad b = 6,5 \end{array}$$

Figure 3. The Answer of a Student in Structural Abstraction Level

Structural Awareness

Based on the result of the students' mathematical abstraction ability test for the level of structural awareness, it appears that female students are already at the fourth level, namely structural awareness. Whereas, the two male students had not yet reached this level. The students still have difficulty in finding the result of linear function, thus the students cannot solve the problems correctly and systematically and the students are able to show the summary of their activities, but the solution is wrong. It shows that female students can solve the problems well. According to the research by Robbins (1998), in females, the smaller the hippocampus, it makes better and it works. The experts are investigating how the hippocampus works and say that the female brain works more effectively and efficiently because the neurons that regulate the females brain communicate better with each other than the neurons in the male brain. Therefore, females have the ability to complete the tasks assigned without having to involve a large number of neurons in the process, and their small size may represent more intense neuronal packing of cells or more active signals in females so they can work more effectively and efficiently in many ways (Mawaddah, Ahmad, & Duskri, 2018). Here is one of the examples of the student answer in structural awareness level the problem.

Jadi: $f(x) = ax + b$
 $f(x) = 2x + 9$
 $f(2) = 2 \cdot 2 + 9 = 4 + 9 = 13$
kesimpulan $f(x) = 2x + 9$
 $m(x) = 13$

Figure 4. The Answer of a Student in Structural Awareness Level

Based on the results of this study, it can be seen that the mathematical abstraction ability of female students reaches the level of structural awareness while for the mathematical abstraction ability of male students only reaches the level of recognition. This means that the mathematical abstraction ability of female students is higher than male students. This fact is in line with Lubienski, Robinson, Crane, & Ganley (2013) research which states that women are believed to have the ability to learn mathematics better than men. In addition, the results of Misu, Hasnawati, & Rahim (2019) study stated that the mathematical abilities of female mathematics education students were higher than male students. Their high abilities are gained due to experience and more practice (Firdiani, Herman, & Hasanah, 2020). This is reinforced by Fitriani, Suryadi, & Darhim (2018) research of a student with high abilities who has useful experience in solving problems.

Conclusion

This study discusses the ability of mathematical abstraction of junior high school students in function seen from the gender. Based on the result and the discussion that has been explained, it shows that some female students are suitable for all abilities of mathematical abstraction. The results of female students fulfill all the abilities of mathematical abstraction. Meanwhile, the results of male students have just achieved the recognition ability, it seems that male students have higher difficulty than female students. The results of male students are lower, especially in the last ability, namely structural awareness (Jupri, Drijvers, & Heuvel-Panhuizen, 2014). The higher level of the mathematical abstraction ability, it can be more complicated for the students' needs. Therefore, learning strategy is needed to build concepts with a student-centered learning approach that can develop mathematical abstraction abilities.

References

- Cifarelli, V. V. (1988). *The Role of Abstraction as a Learning Process in Mathematical Problem Solving* (Doctoral Dissertation). USA: Purdue University.
- Firdiani, N. H., Herman, T., & Hasanah, A. (2020). Gender and Mathematical Communication Ability. *Journal of Physics: Conference Series*, 1512(1), 1–5. <https://doi.org/https://doi.org/10.1088/1742-6596/1521/3/032095>
- Fitriani, N., Suryadi, D., & Darhim, D. (2018). Analysis of Mathematical Abstraction on Concept of a Three Dimensional Figure with Curved Surfaces of Junior High School Students. *Journal of Physics: Conference Series*, 1132(1), 1–7. <https://doi.org/https://doi.org/10.1088/1742-6596/1132/1/012037>

- Fitriani, N., Suryadi, D., & Darhim, D. (2018). The Students' Mathematical Abstraction Ability Through Realistic Mathematics Education with VBA-Microsoft Excel. *Infinity–Journal of Mathematics Education*, 7(2), 123–132. <https://doi.org/https://doi.org/10.22460/infinity.v7i2.p123-132>
- Harding, S. (1986). *The Science Question in Feminism*. Ithaca. New York, NY: Cornell University Press.
- Hershkowitz, R., Schwarz, B., & Dreyfus, T. (2001). Abstraction in Context: Epistemic Actions. *Journal for Research in Mathematics Education*, 32(2), 195–222. <https://doi.org/https://doi.org/10.2307/749673>
- Jupri, A., Drijvers, P., & Heuvel-Panhuizen, M. V. D. (2014). Difficulties in Initial Algebra Learning in Indonesia. *Mathematics Education Research Journal*, 26(4), 683–710. <https://doi.org/https://doi.org/10.1007/s13394-013-0097-0>
- Kant, I. (1855). *The Critique of Pure Reason* (Meiklejohn Translation). London: Henry G. Bohn.
- Komala, E. (2018). Analysis of Students' Mathematical Abstraction Ability by Using Discursive Approach Integrated Peer Instruction of Structure Algebra II. *Infinity–Journal of Mathematics Education*, 7(1), 25–34. <https://doi.org/https://doi.org/10.22460/infinity.v7i1.p25-34>
- Lubienski, S. T., Robinson, J. P., Crane, C. C., & Ganley, C. M. (2013). Girls' and Boys' Mathematics Achievement, Affect, and Experiences: findings from ECLS-K. *Journal for Research in Mathematics Education*, 44(4), 634–645. <https://doi.org/https://doi.org/10.5951/jresematheduc.44.4.0634>
- Mawaddah, Ahmad, A., & Duskri, M. (2018). Gender Differences of Mathematical Critical Thinking Skills of Secondary School Students. *Journal of Physics: Conference Series*, 1088(1), 1–6. <https://doi.org/https://doi.org/10.1088/1742-6596/1088/1/012054>
- Misu, I., Hasnawati, & Rahim, U. (2018). Analysis of Mathematical Ability based on Gender. *Journal of Physics: Conference Series*, 1188(1), 1–5. <https://doi.org/https://doi.org/10.1088/1742-6596/1188/1/012054>
- Mitchelmore, M. C., & White, P. (2007). Abstraction in Mathematics Learning. *Mathematics Education Research Journal*, 19(2), 1–9. <https://doi.org/https://doi.org/10.1007/BF03217452>
- Muniroh, A., Usodo, B., & Subanti, S. (2017). Algebraic form Problem Solving based on Student Abstraction Ability. *Journal of Physics: Conference Series*, 895(1), 1–7. <https://doi.org/https://doi.org/10.1088/1742-6596/895/1/012038>
- Nurhasanah, F., Kusumah, Y. S., & Sabandar, J. (2017). Concept of triangle : Examples of Mathematical Abstraction in Two Different Contexts. *IJEME–International Journal on Emerging Mathematics Education*, 1(1), 53–70. <https://doi.org/http://dx.doi.org/10.12928/ijeme.v1i1.5782>
- Nurhasanah, F., Sabandar, J., & Kusumah, Y. S. (2013). Abstraction Processes in Learning Geometry using GSP. *Proceeding of 6th East Asia Regional Conference on Mathematics Education (EARCOME6)*, 1–9. <https://doi.org/https://doi.org/10.13140/2.1.1452.0005>
- Piaget, J. (1972). *The Principles of Genetic Epistemology*. London: Routledge & Kegan Paul.
- Piaget, J. (2001). *Studies in Reflecting Abstraction*. Philadelphia, PA: Psychology Press.

Putra, J. D., Suryadi, D., & Juandi, D. (2018). Mathematical Abstraction Ability of Prospective Math Teacher Students. *Journal of Physics: Conference Series*, 1132(1), 1–6. <https://doi.org/https://doi.org/10.1088/1742-6596/1132/1/012049>

Robbins, S. P. (1998). *Perilaku Organisasi*. Jakarta: PT. Prenhallindo.

Subroto, T., & Suryadi, D. (2018). Epistemologi Obstacles in Mathematical Abstraction on Abstract Algebra. *Journal of Physics: Conference Series*, 1132(1), 1–6. <https://doi.org/http://doi.org/10.1088/1742-6596/1132/1/012032>

Tall, D. (1991). *Advanced Mathematical Thinking*. London: Kluwer Academic Publishers.

Copyrights

Copyright for this article is retained by the author(s), with first publication rights granted to the journal.

This is an open-access article distributed under the terms and conditions of the Creative Commons Attribution license (<http://creativecommons.org/licenses/by/4.0/>).