



Probabilistic Thinking Level of Junior High School Students with Low Mathematical Abilities Who Have Not Learn Probability Material

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Abstract

This research aims to describe the level of probabilistic thinking of Junior High School of 2 Rembang students with low mathematical abilities and have not formally learn probability material in the sample space construct, experimental probability from an event, and probability comparison. The subjects of this research were 2 low mathematical students of class VII.7. This is a qualitative research using case studies. The main instrument in this research was the researcher, and the supporting instruments in this research were tasks in the form of probabilistic problems and interview guidelines. The data obtained by the researcher was validated using time and source triangulation. The results show that the level of probabilistic thinking of junior high school students with low mathematical abilities who had not formally learn probability material is as follows: (a) In the sample space construct, students are at level 2 (transitional level) with the characteristics of being able to register a complete set of results of a one-stage experiment and sometimes a two-stage experiment. (b) In the construct of experimental probability for an event, students are at the subjective level (level 1) with the characteristics of using subjective opinions to determine the most likely or least likely events. (c) In the probability comparison construct, students are at the subjective level (level 1) with the characteristics using subjective opinion to compare the probability of an event in two different sample spaces.

Keywords: *Probabilistic Thinking; Low Mathematics Ability; Probability*

Introduction

In everyday life, everyone is always faced with various situations. It can be in the form of what has already happened, is happening, or will happen that is not certain to happen, or may not happen, but the situations that are still possible. The situation that will occur is a situation that contains an element of uncertainty. Sujadi (Novitasari 2017) states that the situation that will occur contains an element of uncertainty called a probabilistic situation. In responding to a probabilistic situation, a person performs thinking activities by considering things that can influence whether a situation occurs or not. Lamprianou (2003) states that mental activity associated with contexts that contain elements of uncertainty is called probabilistic thinking.

According to Falk and Konold (Brown, 1996), probabilistic thinking is an inherently new way of processing information as the world view shifts from deterministic view of reality. Another opinion from Hogg and Tanis (Jones et al., 1999) which states that probabilistic thinking will be used to describe students' thinking in responding to various kinds of probabilistic situations. An example of probabilistic thinking is when a student decides to choose a school even though he does not know how likely it is to be accepted into the school. According to Amir and Williams (1999), probabilistic thinking involves formal and informal probability knowledge.

Formal probabilistic knowledge is student knowledge built in academic settings, while informal probabilistic knowledge is student knowledge built in non-academic settings. Culture that includes language, beliefs (religion), and experiences (example: games) affects students' informal knowledge (Jones et al., 1999; Sharma, 2012). This assumes that the ability to think probabilistically already belongs to students who have not formally learned probability material. In fact, the terms of probability such as possible, impossible, and must have been known to students in everyday games such as dice game, Hompimpa game, and so on before they learned probability material.

Probability seeks to measure uncertainty as a tool for making decisions (Kapadia et al., 1991; Greer & Mukhopadaya, 2005; Sharma, 2012; Hokor, 2020). Therefore, in everyday life probability is considered very important. This results in the probability of being included in the mathematics curriculum in most countries in the world (Jones et al., 2007; Ministry of education, 2007; Pratt, 2005; Schield, 2010). Students' thinking has different levels when answering a probabilistic problem (Jun, 2002).

Students' thinking in responding to various probabilistic problems has a different level known as the probabilistic level (Kurniasih & Sujadi, 2017). There are 4 levels of probabilistic thinking, as follows: (a) Level 1 (subjective level) is associated with subjective or non-quantitative thinking. (b) Level 2 (transitional level) is seen as a transition period between subjective thinking and natural quantitative thinking (naïve quantitative). (c) Level 3 (informal quantitative level) is concerned with informal quantitative thinking. (d) Level 4 (numerical level) includes numerical reasoning (Jones et al., 1999).

According to the results of research by Hodnik Cadez (2011), there were 623 students from 6 elementary schools and 1 state kindergarten of Athena in response to probability tasks at level 2 (transition level). The results of research by Mousoulides (2009) show that kindergarten students who have not formally learned about the probability material was successfully solving problems related to several probability concepts. Research results from Jones et al. (1999) show that grade 3 elementary school students who formally learn about probability material in advance with the topic of the sample space, the probability of an event, comparison of probabilities, conditional probabilities, and free probability, and using probability tasks in the context of selecting class leaders, were at level 1 to 4.

Way (2003) stated that students around the age of 9 have basic probability concepts and are most likely to respond to lessons that help them to develop simple numerical strategies into proportional thinking. Based on the research results of Sari et al. (2017) that there is a difference in response to the probability task between high-ability and low-ability grade 5 elementary students. The results of Kurniasih & Sujadi's research (2017) show that 237 of grade 8 junior high school students were at level I and prior to getting learning probability, while after being given learning about the probability level, some students increased to level 4. Meanwhile, the results of Mahyudi's research (2016) in class XI students SMA Negeri 9 Bengkulu City show that there are 3 people at level 1, 16 people at level 2, 4 people are at level 3, and no students at level 4.

The results of both international and national researches indicate that there are differences in students' level of probabilistic thinking between school levels from kindergarten to high school. The

level of probabilistic thinking of junior high school students who informally learn probability is likely to be different from them who formally learn probability. Based on the 2013 revised curriculum in Indonesia, probability material is first studied when students are in grade 8 semester II, whereas outside of school, students often listen to several terms or games that take advantage of the concept of probability. Therefore, researchers are interested in re-examining previous research on the level of probabilistic thinking of students who have not formally learned about probability. This research focuses on students of Junior High School of 2 Rembang who have low mathematical abilities and have not formally learn probability material.

Methodology

This is a qualitative research using case studies. The subjects in this research were 2 students of grade VII of Junior High School of 2 Rembang in the odd semester of the 2020/2021 academic year who had low mathematical abilities. Determination of the research subject was carried out by purposive sampling technique with several criteria as follows: (1) the student has not learned material about probability, (2) based on the classification of high, medium and low initial abilities (2 students each in each category), (3) teacher recommendations and students' abilities in expressing ideas or ideas verbally or in writing for smooth communication between researchers and research subjects.

The main data in this research is information about the probabilistic thinking of SMPN 2 Rembang students who have low mathematical abilities and have not formally learned probability. The main data sources of this research were student answers and student responses during the interview process which were recorded through written notes or through a recording device. The data collection technique was carried out using task-based interviews. The technique of checking the validity of the data in this research used time triangulation and source triangulation.

Results and Discussion

This research was conducted at the junior high school level, especially for students who had not formally learned about probability, so the researchers used three main constructs of the probabilistic thinking level framework, namely the sample space, the experimental probability of an event, and the comparison of probabilities. In describing and analyzing the data, the researcher used several initials to facilitate the data analysis process, namely (a) the researcher was symbolized by P. (b) the subject was symbolized by TKARX, where X was the order of the subject {1,2}.

The level of students' probabilistic thinking in solving probabilistic problems in this study adopts the 4 levels proposed by Jones et, al. (1999) based on the sample space construct, the probability of an event, and the comparison of probabilities. The following shows the results of the data analysis of the TKAR1 and TKAR2 subjects in completing task I and task II for each construct.

Sample Space Construct

Table 1 and table 2 below describe the result of summary of interview data analysis based on Task 1 and II in sample space construct.

Table 1. The Analysis Result of Interview Stage 1 and 2 with the subjects of TKAR1 in Sample Space Construct

| Experiment | Interview Stage I | Interview Stage II |
|--|---|---|
| One-Stage | Subjects can fully register the results of a one-stage experiment. This is indicated by mentioning all the dice contained in the dice. | Subjects can fully register the results of a one-stage experiment. This is indicated by mentioning all the dice contained in the dice. |
| Two-Stage | Subjects can register a complete list of all the results of the two-stage experiment. This is indicated by listing all the color pairs, even though at first the subject did not completely list all the color pairs. | Subjects can register a complete list of all the results of the two-stage experiment. This is shown by listing all the number pairs, even though at first the subject did not completely list all the number pairs. |
| The valid data of subject KTAR1 is as follows: One-Stage Experiment Subject is consistently can register completely the one stage experiment. Two-Stage Experiment Subject can register completely the two stage experiment. | | |

Table 2. The Result of Interview Analysis Stage 1 and II with the subject of TKAR2 in Sample Space Construct

| Experiment | Interview Stage I | Interview Stage II |
|--|---|---|
| One-Stage | Subjects can fully register the results of a one-stage experiment. This is indicated by mentioning all the dies contained in the dice. | Subjects can fully register the results of a one-stage experiment. This is indicated by mentioning all the dies contained in the dice. |
| Two-Stage | Subjects can register a complete list of all the results of the two-stage experiment. This is indicated by listing all the color pairs. | Subjects can register a complete list of all the results of the two-stage experiment. This is indicated by listing all the color pairs. |
| The valid data of subject KTAR2 is as follows: One-Stage Experiment Subject can consistently register completely the result of one-stage experiment. Two-Stage Experiment Subject can consistently register completely the result of two-stage experiment. | | |

Based on the triangulation of the results of the interview data analysis based on task I and II of the TKAR1 and TKAR2 subjects above, there are similarities in the results in data collection stages I and II in the sample space construction. The same data is valid, while different data is used as another finding in the research. Based on the results of the data analysis of the level of probabilistic thinking, it appears that in the construct of the experimental sample space at one level the two subjects can register completely all members of the sample space. Subjects TKARI and TKAR2 listed all possible dice to get when rolling a dice once.

In a two-stage experiment, both subjects were able to register a complete sample space member. This can be seen from the answers to tasks and interviews of the two subjects who were able to completely mention all the color pairs that appeared from spinning tops A and B in task I as well as all number pairs that appeared from spinning tops A and B on task II. Although initially TKAR1 subjects were still doubtful and confused when collecting data in stage I. When researchers asked questions about how to register color and number pairs in data collection stages I and II, the two subjects could not show how to register pairs. This shows that the two subjects cannot use generative strategies. Based on these results, it appears that in the construct of the sample space the subjects TKAR1 and TKAR2 are at level 2

(transitional level). Both TKAR1 and TKAR2 subjects can register a complete set of results for one-stage experiments and sometimes for two-stage experiments.

The Construct of Experimental Probability from an Event

Table 3 and table 4 describe the summary of the result of interview analysis based on the problem of probabilistic I and II with the subject of TKAR1 and TKAR2 in experimental probability construct of an event.

Table 3. The Result of Interview Analysis Stage I and II of Subject TKAR1 in Experimental Probability Construct from an Event

| Experiment | Interview Stage I | Interview Stage II |
|---|---|--|
| One-Stage | Subjects use subjective opinions to determine the most likely events. The subject answers the dice less than 4 which has the most probability of rolling a dice. Subject uses subjective opinion to determine the approximate size of getting an even die. Subjects answered between 2, 4, and 6 for the estimated appearance of the even die. | Subject uses opinion to determine the most likely event. The subject answers the dice more than 2 which has the most probability of being obtained when rolling a dice. Subject uses subjective opinion to determine the likelihood of getting odd die. Subjects answered 1, 3, and 5 for the most likely odd dice to appear. |
| Two-Stage | Subject can find the most likely event. Subjects answering top A and top B may stop at the same color because they have the same color. | Subject can find the most likely event. Subjects answered top A and top B might both stop at number 2 because in the two tops there is a number 2. |
| The valid data of subject KTAR1 is as follows: Subjects can determine the most likely events for one-stage and two-stage experiments. The subject cannot estimate the likely events for a one-stage experiment. | | |

Table 4. The Result of Interview Analysis of Stage I and II with Subject TKAR2 in Experimental Probability Construct from an Event

| Experiment | Interview Stage I | Interview Stage II |
|---|---|---|
| One-Stage | Subjects use subjective opinions to determine the most likely events. The subject answers the dice less than 4 which has the most probability of rolling a dice. The subject reasoned that there are 3 dice that are less than 4, while the dice that is more than 4 are 2. Subject cannot determine the approximate size of getting even die. Subjects answered 50% for the estimated appearance of even dice, the subject thought that because of the 6 dice, it could be that the dice were not even. | Subjects use subjective opinions to determine the most likely events. Subjects answer the dice more than 2 which have the most probability of being obtained when rolling a dice. Subject cannot determine the probability of getting odd die. Subjects answered 50% for the most likely odd dice, the subject argued that because dice are 6, they can get dice that are not odd. |
| Dua Tingkat | Subject can find the most likely occurrence. Subjects answering top A and top B may stop at the same color because they have the same color. | Subject can find the most likely occurrence. Subjects answered top A and top B might both stop at number 2 because in the two tops there is a number 2. |
| The valid data of KTAR2 is as follows: - Subjects can determine the most likely events for one-stage and two-stage experiments. - The subject cannot estimate the likely events for a one-stage experiment. | | |

Based on the triangulation of the results of the interview data analysis based on task I and II from the TKAR1 and TKAR2 subjects above, it appears that there are similarities in the results in data collection stages I and II on the experimental probability construct of an event. The same data is valid data so that it is used as the main finding, while different data is used as another finding in the research. Based on the results of data analysis of the level of probabilistic thinking, as presented in Table 3 and Table 4, the TKAR1 and TKAR2 subjects can determine the most likely events for one-stage and two-stage experiments using subjective opinions. This can be seen from the answers of the TKAR1 and TKAR2 subjects in the experimental probability of a one-stage event which states that the dice less than 4 have the most probability of getting a die when rolling a dice because the die that is less than 4 are 4 while the dice that is more than 4 there are 2. The reasons that are disclosed are imprecise and subjective. In addition, the TKAR1 and TKAR2 subjects were also unable to estimate the likelihood for one-stage experiments. Actually the TKAR2 subject when answering it is most likely that the one level experiment is correct, but not quite right. The answer should be using fractions because what is being asked is the experimental probability or probability of an event being asked.

In this construct, the subjects TKAR1 and TKAR2 did not realize that a wider sampling was needed to determine the most likely events. This can be seen when the two subjects answered disagree when the researcher thinks the probability of getting the number 4 which appears 6 times is greater than the probability of getting the number 5 which appears 8 times. They assume that the numbers that appear the most have ample probabilities to appear again. Based on these results, it appears that in the experimental probability construct an incident TKAR1 and TKAR2 subjects are at level 1 (subjective level). Both TKAR1 and TKAR2 subjects were able to determine the most likely occurrence for both one-stage and two-stage experiments.

Probability Comparison Construct

In Table 5 and Table 6 below, a summary of the results of the analysis of the task-based interview with probabilistic problems I and II TKAR1 and TKAR2 is presented in the probability comparison construct.

Table 5. The Results of Interview Analysis Stage I and II Subject TKAR1 in the Construct Probability Comparison

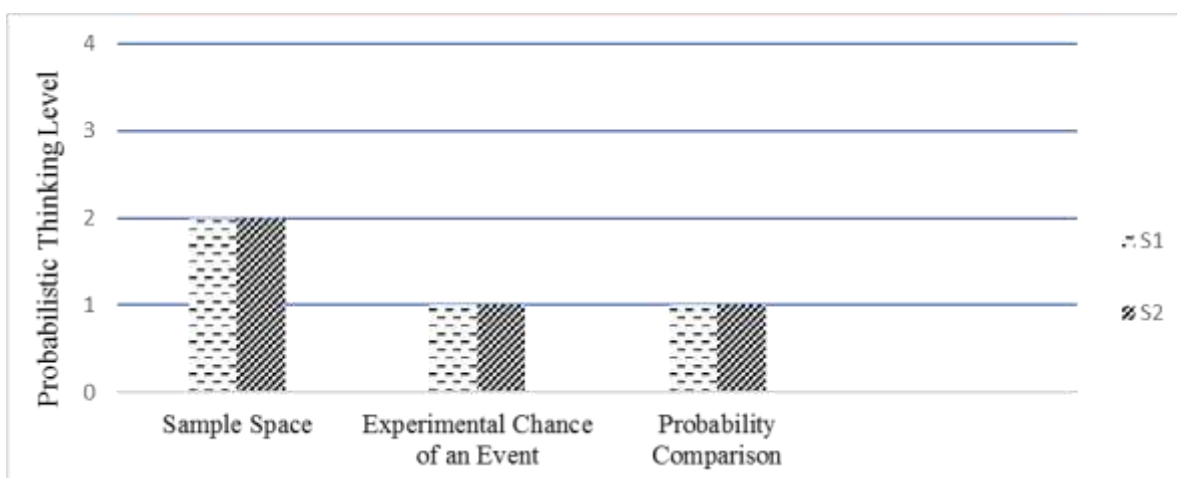
| Experiment | Interview Stage I | Interview Stage II |
|---|---|--|
| One-Stage | Subjects cannot determine the numerical value of the probability of getting dice less than 4. Subjects answered 80% on the grounds that there are four dice that are less than 4 which are more numerous than the number of dice less than 4 which are only two dice. | The subject cannot determine the numerical value of the probability of getting dice more than 2. The subject answered 80% on the grounds that the number of dice that is more than 2 is more than the number of dice that is less than 2. |
| Two-Stage | Subjects cannot determine numerical values on a two-stage experiment. The subject answered that the probability of each top to get yellow was 85%, with the reason that each top had a yellow color. | Subjects answered most likely that top A gets the number 3 is 95% on the grounds that top A has 4 digits so the probability is small. While the probability of top B to get the number 1 is 75% because top B has 1 number so the probability is high. |
| The valid data of subject is as follows: Subject cannot determine the numerical value toward the one-stage and-two stage experiment. | | |

Table 6. The Result of Interview Analysis Stage I and II of Subject TKAR2 in Probability Comparison Construct

| Experiment | Interview Stage I | Interview Stage II |
|---|--|--|
| One-Stage | Subjects cannot determine numerical values on a two-stage experiment. The subject answered that the possibility of each top getting the number 1 was 80% on the grounds that the two tops had a 1, so the possibility of getting the number 1. | Subjects cannot determine the numerical value of the probability of getting dice more than 2. Subjects answered that the most likely number of dice is 80%. |
| Two-Stage | Subjects cannot determine numerical values on a two-stage experiment. Subjects answered that the probability of each top to get yellow is 25% because there are 3 colors besides yellow. | Subjects cannot determine numerical values on a two-stage experiment. The subject answered that the possibility of each top getting the number 1 was 80% on the grounds that the two tops had a 1, so the possibility of getting the number 1. |
| The valid data of KTAR2 is as follows: Subjects cannot determine the numerical value of one-stage and two-stage experiment | | |

Based on the triangulation of the results of the interview data analysis based on task I and II of the TKAR1 and TKAR2 subjects above, it is seen that in determining the numerical value, the two subjects answered incorrectly and used subjective reasons. In this case, the TKAR1 and TKAR2 subjects began to distinguish between fair and unfair probabilities. Both of them think that the probability of a yellow top which has four sides are not fair when compared to a top that has 3 sides. But in answering questions, sometimes TKAR1 and TKAR2 subjects still use subjective opinions when comparing the probability of occurrence of two different sample space.

Based on the results of the analysis, it appears that the TKAR1 and TKAR2 subjects for the probability comparison construct are at level 1 (subjective level). Both TKAR1 and TKAR2 subjects were unable to determine numerical values for the one-stage and two-stage experiments. Both subjects use subjective opinions in determining the experimental probability of an event. The level of probabilistic thinking of the TKAR1 and TKAR2 subjects in each construct is presented in Figure 1 below.



The level of probabilistic thinking of Junior High School of 2 Rembang students with low mathematical ability and formally has not learned probabilities in the sample space construct is at level 2 (transitional level). Subjects with low mathematical abilities can register a complete set of experimental

results at one level and sometimes for two-stage experiments. In the experimental probability construction, an event of low-ability subjects is at level 1 (subjective level). In this case the subject with low mathematical ability can determine the most likely events for one-stage and two-stage experiments. In the construct, the comparison of probabilities for low-skilled subjects is at level 1 (subjective level). In this case the subject with low mathematical ability cannot determine the numerical value of the one-stage and two-stage experiments. This is in accordance with the results of research by Kurniasih and Sujadi (2017) which explains that grade 8 junior high school students aged 13-15 years before getting learning probabilities are at the subjective and transitional levels (levels 1 and 2).

Conclusion

The results of the analysis above indicate that the level of probabilistic thinking of Junior High School of 2 Rembang students with low mathematical abilities who have not formally received learning probability is as follows: (a) In the sample space construct, students are at level 2 (transitional level) with the characteristics of being able to register a set complete one-stage experimental results and sometimes for two-stage experiments. (b) In the construct of experimental probability for an event, students are at the subjective level (level 1) with the characteristics of using subjective opinions to determine the most likely or least likely events. (c) In the probability comparison construct, students are at the subjective level (level 1) with characteristics using subjective opinion to compare the probability of an event in two different sample spaces.

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