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Mathematical Modeling of Ballistic Missile Defense

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Abstract

Today, many countries are looking for missile strikes to achieve their goals. To optimize ballistic missile defense, missile defense centers identify potential launch points for enemy missiles to anticipate enemy attacks to reduce potential damage. One of these measures is mathematical modeling for the scenario of a possible enemy attack and defensive cover against this attack. In this research, mathematical optimization and mixed integer linear program have been used to reduce the damage against the enemy attack. The purpose of this study is to minimize the maximum damage caused by enemy missile attacks.

Keywords: *Ballistic Missile Defense; Missile Attack; Mixed-Integer Linear Program; Mathematical optimization*

Introduction

War is one of the most important social phenomena. It seems that since man entered the world, war has been his companion and companion. Very few historical passages can be found in which there is no bloodshed and human violence, so war has occupied the human mind for many years and centuries, and the question has been, what is war? Why does it occur? What effects does it have on the political, social and psychological issues of the people of the society? And what are the types of war? (Moradi, 2003, P. 5). A very famous quote from Carl von Clausewitz states: "War is a continuation of politics, but in a different way and with a different language" (Gary, 1999, P. 13). In future modern warfare, the role of missiles and rockets will be far greater than in the past, except that in the past this type of weapon was used more for mass fire, and in the future missiles and missiles can hit tactical targets from a distance. (Tajerian, 2009, P. 7).

The great world powers have been trying for decades to prevent the proliferation of nuclear weapons. Their efforts have led to the limitation of nuclear capabilities to several countries and the creation of an international non-proliferation regime, which has focused on the Nuclear Non-Proliferation Treaty since 1968. Recently, however, international attention has shifted to the development of nuclear weapons systems, which are being considered separately from nuclear warheads themselves. The proliferation of ballistic missile systems and technologies in areas such as the Middle East, where intense regional tensions prevail, has raised concerns. The widespread use of ballistic missiles in the 1988 Iran-Iraq civil war, and Iraq's use of Scud missiles against Israel and Saudi Arabia during Operation Desert Storm has raised the issue that such systems could threaten regional stability and upset regional balance. These signs indicate what will happen if developing countries arm ballistic missiles with weapons of mass destruction (Hosseini and Karami, 1998, Pp. 409-410).

Figure1. Scud ballistic missile (Tasnim News Agency Defense Correspondent, 2015)

Statement of issue

The military thinker Carl von Clausewitz, who is considered by some to be the greatest philosopher of war, says: "Everything seems simple in war, but even the simplest thing is difficult in war" (Hatami, 2003, P. 131). "Plans may be worthless in themselves, but everything is summed up in planning," says Dwight D. Eisenhower, an American politician and politician (Gray, 1999, P. 170). Operations research science, as its name suggests, refers to military operations research, which indicates multiple goals, resource constraints, and complex relationships between variables (Ajorlu, 2010, P. 3). Ballistic missiles are capable of carrying high-explosive missile warheads, as well as nuclear, microbial and chemical warheads, which are called weapons of mass destruction. Today, various countries are developing defense interceptions and defensive tactics to defeat incoming ballistic missiles (Diehl, 2004).

In this article, it is examined that the two forces of attacker (enemy) and defender (internal force) are facing each other. The attacker's goal is to launch which missiles from which position to which targets in order to maximize the total of expected value (average) of the damage to the target. The goal of the defender is how to deploy in a predetermined location (pre-deployment) and select the best defensive platform according to the enemy strategy. Therefore, the defending operation is air interception of the attack and choosing the best defense option to deal with enemy missiles and minimizing the damage to the targets. In general, the enemy plan is to maximize the damage and the defense plan is to minimize the enemy damage. This problem becomes a two-way game that is solved after becoming a one-level model.

Introducing the indicators

Attacker:

l∈L: The location of the ballistic missile launch by the attacker.

m∈M: Attack missile type.

t∈T: Attacker (target Defender).

a∈A: Attack (a missile attack aimed at a target).

$a \in A_{m,l}$: Attack with m type missile from the launch site l.

$a \in A_t$: Attack on target t.

$l_a \in L$: The location of the attack a.

$m_a \in M$: The type of missile launched in the attack a.

$t_a \in T$: The target of the attack a.

Defender:

$p \in P$: Defensive platform.

$c \in C$: Class (type) of defense platform.

$c_p \in C$: Class (type) of defense platform p.

$g \in G$: Geographical location of the defense platform.

$g_c \in G_c \subseteq G$: Geographical location of the defense platform of class c.

$i \in I$: Types of defense interceptors.

$d \in D$: Defensive option.

Data (Units)

Attacker:

$\overline{mob}_m$: Total inventory of mobile missiles type m.

$\overline{fix}_{m,l}$: Total inventory of fixed type m missiles at launch site l.

$\underline{mov}_{m,l}$: Minimum number of m-type mobile missiles that the attacker can deliver to the launch site.

$\overline{mov}_{m,l}$: Maximum number of mobile missiles that the attacker can deliver to the launch site.

$\overline{Max\_m}_t$: Maximum number of missiles that can hit target t.

V_t: Target value t (values)

k_a: Probability that attack a will not be intercepted and hit the target t_a and destroy it (probability of destroying target t_a).

Defender:

$LO_{p,i}$: Number of Type I interceptor missiles carried by the p-platform (interceptors).

$S_{a,c,d,i}$: Number of interceptor missiles used by Class C platform with defense option d against attack a.

$\overline{shoot_p}$: Maximum number of interceptor missiles on the p-platform that can be fired at the enemy in one operation.

$Pk_{a,c,g,d}$: Possibility of intercepting and neutralizing attack a by a class c platform in geographical location g by applying the defense option d.

Variables

Attacker:

$W_{m,l}$: Number of m-type mobile missiles delivered to the launch site l.

Y_a: If the attack is 1, otherwise it is 0.

Defender:

$X_{p,g}$: If the p platform is in geographical position g is 1 and otherwise it is 0.

$R_{a,p,g,d}$: If the attack a with the p platform in the geographical position $g \in G_{cp}$ is involved with the application of the defense option d is 1, otherwise it is 0.

Mathematical model of the problem

Brun et al. (2005) stated the problem of P2 optimization as follows. In this case, the maximum mathematical expectation of enemy damage is minimized by the defender, which is in the form of a minimum.

$P2:$

$$Z^* = \min_{\{X,R\} \in XR} \left[\begin{array}{llll} \max\limits_{Y} \sum\limits_{t \in T} V_t \sum\limits_{a \in A_t} k_a \left(1 - \sum\limits_{p,g,d} Pk_{a,c_p,g,d} R_{a,p,g,d} \right) Y_a & & & (1) \\ s.t. \quad \sum\limits_{l \in L} W_{m,l} \le mob_m, & \forall m \in M, & [\alpha_m] & (2) \\ -W_{m,l} + \sum\limits_{a \in A_{m,l}} Y_a \le fix_{m,l}, & \forall l \in L, m \in M, & [\beta_{m,l}] & (3) \\ \sum\limits_{a \in A_t} Y_a \le \overline{Max\_M}_t, & \forall t \in T, & [\gamma_t] & (4) \\ \underline{mov}_{m,l} \le W_{m,l} \le \overline{mov}_{m,l}, & \forall l \in L, m \in M, & [-\underline{\pi}_{m,l}, \overline{\pi}_{m,l}] & (5) \\ Y_a \in \{0,1\}, & \forall a \in A. & [\theta_a] & (6) \end{array} \right]$$

The variables written in parentheses to the right of each constraint represent the dual variable of that constraint, which we will use later. In the case of internal maximization, the attacker uses the objective function (1) to maximize the mathematical expectation of damage to the target. The limit (2) indicates the number of mobile missiles of any type that can be delivered to the entire launch site. The limit (3) indicates the number of mobile and stationary missiles of any type that can be launched from any location. The limit (4) indicates the number of missiles that can be fired at any target. The limit (5) The limit on the number of mobile missiles of any type that can be delivered to any launch site. The limit (6) Specifies the type of the variable Y_a. In fact, the calculated value of the objective function (1) for a large area such as an airport is a standard cumulative model and is calculated from the sum of several attacks. This means that one attack may not be able to destroy an area and multiple attacks are used, while a small target point may be destroyed by a missile attack, and if the attacks continue at that point, it can cause excessive damage to the targets. To solve this problem, a limit (4) is introduced, which states that the attacker can hit the target at most once.

External constraints The problem of minimizing the defender

The external constraints of the defender minimization problem are as follows:

$$\sum_{g \in G} X_{p,g} \leq 1, \qquad \forall p \in P, \qquad (7)$$

$$\sum_{p \in P} X_{p,g} \leq 1, \qquad \forall g \in G, \qquad (8)$$

$$\sum_{p \in P, g \in G, d \in D} R_{a,p,g,d} \leq 1, \qquad \forall a \in A, \qquad (9)$$

$$\sum_{a \in A, d \in D} S_{a,c_p,d,i} R_{a,p,g,d} - LO_{c_p,i} X_{p,g} \leq 0, \qquad \forall p \in P, i \in I, g \in G_{c_p}, \qquad (10)$$

$$\sum_{a \in A, g \in G, d \in D, i \in I} S_{a,c_p,d,i} R_{a,p,g,d} \leq \overline{shoot}_p, \qquad \forall p \in P, \qquad (11)$$

$$\sum_{d \in D} R_{a,p,g,d} \leq X_{p,g}, \qquad \forall a \in A, p \in P, g \in G, \qquad (12)$$

$$X_{p,g}, R_{a,p,g,d} \in \{0,1\}. \qquad (13)$$

The limit (7) each platform to occupy a maximum of one geolocation position. The limit (8) states that a maximum of one platform can be located in each geographical location. The limit (9) allows a maximum of one interception per attack. In fact, this restriction states that there is no need for one defense for each missile attack, and for each missile attack there is a maximum of one interception, because if the defense is broken, it may be impossible to intercept any attack and this possibility should be allowed. The limit (10) states the limitation of interceptor missiles from any platform and any geographic location, meaning that the missiles used against an attack must be less than the total number of missiles carried by the platform. The limit (11) makes interceptor missiles fired from each platform less than the maximum number of missiles that that platform is allowed to fire. In the limit (12), a maximum of one conflict can occur from each platform located in a geographic network. The limit (13) specifies the type of decision variables.

An integer linear program for minimizing maximum damage

In order to solve the mini-max model, a simpler specialized analysis algorithm can be created. The decision variable W, which is the number of moving missiles that can be delivered to the launch site, is an integer. On the other hand, the variable Y, which is the attack strategy, is of type zero and one. The coefficient matrix of the model is a one-time matrix, and since all the data to the right of the number are integers, then all the principal solutions of the liberated linear programming problem of the invasive maximization problem are inherently integers. Using the dual variables introduced in Model P2, we take the internal dual maximization model and create a "min-min" problem. Finally, the linear programming model of the MILP2 mixed integer is obtained. The answer to the linear programming problem of integer number determines the optimal pre-position value of X* and the execution of interception R*. By placing the fixed values X = X* and R = R* in model P2, a linear programming is created and the optimal design value The transfer of W* mobile missiles and the optimal Y* attack plan are achieved. It is important to note that in all cases, the matrix constraint is a one-time attacker maximization problem. The two problems of internal maximization P2 are as follows:

$$DP2:$$

$$\min_{\alpha,\beta,\gamma,\pi,\theta,R} \sum_{m\in M} mob_m \alpha_m + \sum_{m\in M,l\in L} fix_{m,l}\beta_{m,l} + \sum_{t\in T} \overline{Max\_M}_t \gamma_t - \sum_{m\in M,l\in L} \underline{mov}_{m,l}\underline{\pi}_{m,l} + \sum_{m\in M,l\in L} \overline{mov}_{m,l}\overline{\pi}_{m,l} + \sum_{a\in A}\theta_a \quad (14)$$

$$s.t.\quad \alpha_m - \beta_{m,l} - \underline{\pi}_{m,l} + \overline{\pi}_{m.l} \geq 0, \qquad \forall m\in M, l\in L, \quad (15)$$

$$\beta_{m_a,l_a} + \gamma_{t_a} + \theta_a + \sum_{p\in P,g\in G,d\in D} k_a V_{t_a} Pk_{a,c_p,g,d} R_{a,p,g.d} \geq k_a V_{t_a}, \qquad \forall a\in A, \quad (16)$$

$$all\quad \alpha_m, \beta_{m,l}, \gamma_t, \underline{\pi}_{m,l}, \overline{\pi}_{m,l}, \theta_a \geq 0. \quad (17)$$

By placing the double part of the maximization of problem P2 (model DP2) in problem P2, the following model is obtained:

$$\min_{X,R\in XR} \left[\begin{array}{ll} \min_{\alpha,\beta,\gamma,\pi,\theta,R} \sum_{m\in M} mob_m \alpha_m + \sum_{m\in M,l\in L} fix_{m,l}\beta_{m,l} + \sum_{t\in T} \overline{Max\_M}_t \gamma_t & \\ \quad - \sum_{m\in M,l\in L} \underline{mov}_{m,l}\underline{\pi}_{m,l} + \sum_{m\in M,l\in L} \overline{mov}_{m,l}\overline{\pi}_{m,l} + \sum_{a\in A}\theta_a & (18) \\ s.t.\quad \alpha_m - \beta_{m,l} - \underline{\pi}_{m,l} + \overline{\pi}_{m.l} \geq 0, \qquad \forall m\in M, l\in L, & (19) \\ \beta_{m_a,l_a} + \gamma_{t_a} + \theta_a + \sum_{p\in P,g\in G,d\in D} k_a V_{t_a} Pk_{a,c_p,g,d} R_{a,p,g.d} \geq k_a V_{t_a}, \qquad \forall a\in A, & (20) \\ \alpha_m, \beta_{m,l}, \gamma_t, \underline{\pi}_{m,l}, \overline{\pi}_{m,l}, \theta_a \geq 0. & (21) \end{array} \right]$$

The objective function of the P2 maximization problem is nonlinear and includes a binary variable (zeros and ones) R. By taking the dual, the maximization model P2 goes out of the nonlinear state and also the binary variable R goes to the right of the constraint (16) and thus the dual model objective function (DP2) will not include the variable R. Since the objective function is a continuous problem, we can combine the halves of the P2 and DP2 models and arrive at the final MILP2 model:

$$MILP2:$$

$$\min_{\substack{\alpha,\beta,\gamma,\pi,\theta \\ X,R}} \sum_{m \in M} mob_m \alpha_m + \sum_{m \in M, l \in L} fix_{m,l} \beta_{m,l} + \sum_{t \in T} \overline{Max\_M}_t \gamma_t - \sum_{m \in M, l \in L} \underline{mov}_{m,l} \underline{\pi}_{m,l} + \sum_{m \in M, l \in L} \overline{mov}_{m,l} \overline{\pi}_{m,l} + \sum_{a \in A} \theta_a \quad (22)$$

$$s.t. \quad \alpha_m - \beta_{m,l} - \underline{\pi}_{m,l} + \overline{\pi}_{m.l} \geq 0, \qquad \forall m \in M, l \in L, \quad (23)$$

$$\beta_{m_a,l_a} + \gamma_{t_a} + \theta_a + \sum_{p \in P, g \in G, d \in D} k_a V_{t_a} Pk_{a,c_p,g,d} R_{a,p,g.d} \geq k_a V_{t_a}, \qquad \forall a \in A, \quad (24)$$

$$\sum_{g \in G} X_{p,g} \leq 1, \qquad \forall p \in P, \quad (25)$$

$$\sum_{p \in P} X_{p,g} \leq 1, \qquad \forall g \in G, \quad (26)$$

$$\sum_{p \in P, g \in G, d \in D} R_{a,p,g,d} \leq 1, \qquad \forall a \in A, \quad (27)$$

$$\sum_{a \in A, d \in D} S_{a,c_p,d,i} R_{a,p,g,d} - LO_{c_p,i} X_{p,g} \leq 0, \qquad \forall p \in P, i \in I, g \in G_{c_p}, \quad (28)$$

$$\sum_{a \in A, g \in G, d \in D, i \in I} S_{a,c_p,d,i} R_{a,p,g,d} \leq \overline{shoot}_p, \qquad \forall p \in P, \quad (29)$$

$$\sum_{d \in D} R_{a,p,g,d} \leq X_{p,g}, \qquad \forall a \in A, p \in P, g \in G, \quad (30)$$

$$\alpha_m, \beta_{m,l}, \gamma_t, \underline{\pi}_{m,l}, \overline{\pi}_{m,l}, \theta_a \geq 0, \quad (31)$$

$$X_{p,g}, R_{a,p,g,d} \in \{0,1\}. \quad (32)$$

The value of concealment

The value of defending concealment:

Using the following method, the advantage of concealing defense platforms is evaluated (Brun et al., 2005). To do this, defense platforms are divided into two parts: open platforms (P∈SEEN) and hidden platforms (P∈SECRET).

$$p \in P = SEEN \bigcup SECRET \qquad , \qquad SEEN \bigcap SECRET = \varnothing \quad (33)$$

This model is based on model (P2) written for hidden platforms.

$S2:$

$$\min_{\{X,R\}\in SECRET} \sum_t V_t \sum_{a\in A_t \mid \left(Y_a^*=1 \wedge \sum_{p\in SEEN,g\in G,d\in D} R_{a,p,g,d}=0\right)} k_a \times \left(\left[1-\sum_{p\in SECRET,g\in G,d\in D} Pk_{a,c_p,g,d} R_{a,p,g,d}\right]\right) \quad (34)$$

$$\sum_{g\in G} X_{p,g} \le 1, \qquad \forall p \in SECRET, \quad (35)$$

$$\sum_{p\in P} X_{p,g} \le 1-\sum_{p\in SEEN} x^*_{p,g}, \qquad \forall g \in G, \quad (36)$$

$$\sum_{p\in SECRET,g\in G,d\in D} R_{a,p,g,d} \le 1, \qquad \forall a \in A, \quad (37)$$

$$\sum_{a\in A,d\in D} S_{a,c_p,d,i} R_{a,p,g,d} - LO_{c_p,i} X_{p,g} \le 0, \qquad \forall p \in SECRET, i\in I, g\in G_{c_p}, \quad (38)$$

$$\sum_{a\in A,g\in G,d\in D,i\in I} S_{a,c_p,d,i} R_{a,p,g,d} \le \overline{shoot}_p, \qquad \forall p \in SECRET, \quad (39)$$

$$\sum_{d\in D} R_{a,p,g,d} \le X_{p,g}, \qquad \forall a\in A, p\in SECRET, g\in G, \quad (40)$$

$$X_{p,g}, R_{a,p,g,d} \in \{0,1\}, \quad (41)$$

The value of attacker concealment:

The following method evaluates the advantage of the attacker being able to hide a subset of its moving missiles from the defender (Brun et al., 2005). To do this, the attack missiles are divided into two parts, the fixed and visible missiles (m∈SEEN) and the hidden moving missiles (m∈SECRET), and their common ground is empty.

$$m \in M = SEEN \bigcup SECRET \quad , \quad SEEN \bigcap SECRET = \varnothing \quad (42)$$

Examples of North Korea and Japan:

In this section, due to the lack of access to information of countries, the information, images and results in the article by Brun et al. (2005) in the scenario of North Korea (enemy) and Japan (defender) are examined, which obtained numbers are obtained by directly solving the MILP2 model by Gomez software. In the following, we will report the numerical results obtained from random data. Figure (2) shows the political map of Japan, North and South Korea.

Figure2: Political map of Japan, North and South Korea

Location of attack missiles:

In this scenario, it is assumed that some of North Korea's ballistic missiles are stationed at launch sites, and some are transferable to specific launch sites. Figure (3) shows the approximate location map and Table (1) shows the launch site of North Korea's ballistic missiles, which are classified according to the country's facilities and bases.

Figure3: Location of North Korean (enemy) ballistic missiles in the shape of a rhombus (Brun et al., 2005).

Table (1) shows the launch location of North Korean ballistic missiles and their geographical coordinates and the number of these missiles at a fixed location. If the missiles are allowed to move to specific launch sites, the same number of stationary missiles, Scud-B, Scud-C and No-Dong mobile missiles are available.

Table 1: Location of North Korean ballistic missiles, geographical coordinates and number of these missiles at a fixed location (Brun et al., 2005).

Throwing place	Longitude	Latitude	Scud-B	Scud-C	No-Dong	Taep'o-Dong I	Taep'o-Dong II
Chiha-ir	38°37′	126°41′	15	20	10	-	-
Chunggang-up	41°46′	126°53′	-	10	10	-	-
Kaggamchan	40°24′	125°12′	-	15	10	-	-
Kanggye	40°07′	126°35′	-	15	10	-	-
Mari'gyongdae	38°59′	125°40′	10	20	10	-	-
Mayang	40°00′	128°11′	-	15	20	-	-
Namgung-ir	39°08′	125°46′	5	15	2	1	1
No-dong	40°50′	129°40′	-	5	15	1	1
Ok'pyong	39°17′	127°18′	15	15	10	-	-
Paegun	39°58′	124°35′	-	15	10	-	-
Pyongyang	39°00′	125°45′	15	15	10	-	-
Sangwon	38°50′	126°05′	15	20	10	-	-
Sunchon	39°25′	125°55′	5	15	10	-	-
Tokch'on	39°45′	126°15′	5	15	15	-	-
Toksong	40°25′	128°10′	5	15	15	-	-
Yong-dong	41°59′	129°58′	-	-	20	1	1

Attack missiles:

Table (2) shows the characteristics of North Korean ballistic missiles with minimum and maximum range. Scud-B, Scud-C and No-Dong missiles are operational, and Taep'o-Dong I and Taep'o-Dong II long-range missiles are being developed. It is assumed that each enemy missile hits the target and destroys it after firing if not intercepted. That is, the probability of the enemy missile hitting the target (k_a) if not intercepted is equal to one, which indicates the worst situation for defense.

Table 2: Minimum and maximum range of North Korean ballistic missiles (Brun et al., 2005).

Ballistic missile	Minimum range (km)	Maximum range (km)
Scud-B	40	330
Scud-C	40	700
No-Dong	1350	1500
Taep'o-Dong I	2200	2900
Taep'o-Dong II	3500	4300

Offensive targets in the list of defense assets

Brun et al. (2005) calculated the value of goals based on the following four factors:

Criticality

The degree of necessity of each of the defending assets is called sensitivity and is denoted by c_t, for example, a sensitive geographical location or location that is important for the success of the operation (Roshan and Farhadian, 2006, p. 246). A high value indicates the high importance and sensitivity of the region and a low value indicates the low importance of the region.

Vulnerability

The sensitivity of a force to the knowledge of the enemy is called vulnerability and is denoted by v_t; Mobility and offensive operations significantly cause vulnerability of units (Roshan and Farhadian, 2006, p. 6). High value indicates high vulnerability and low value indicates low vulnerability.

Reconstitutability/ Recovery/ Repair

The ability of the target to repair and repair the damage in a given period of time is called reconstruction and recovery, which includes the reconstruction of equipment and manpower to the initial state of operation and is indicated by r_t. A high value indicates a long time to improve troops and equipment to the initial state of operation, and a low value indicates the opposite.

Threat

It means the potential for security breaches and is indicated by h_t. The term is synonymous with potential aggression. Invasion means any activity that leads to destruction and threat (Roshan and Farhadian, 2006, p. 63). In this part, the threat means estimating the possibility of the enemy attacking the assets. A high value indicates that the enemy is very likely to attack the target, and a low value indicates the opposite. In Table 3, the enemy targets, which are the same as the defending assets, are given with the geographical coordinates and the value of the targets obtained from Equation (43). Figure 3 shows the location of the targets in a circle on a map of the area. Brun et al. (2005) calculated the target value of t using Equation (43).

$$V_t = val_t = \ln(c_t \times v_t \times r_t \times h_t) + 1 \qquad (43)$$

As can be seen from Equation (43), the value of the target is directly related to its sensitivity, vulnerability, reconstruction and recovery, and threat. For example, the target value of Seoul is with the characteristics (c, v, r, h) = (4, 8, 5, 9), which according to relation (43) Is equal to $\ln(4 \times 8 \times 5 \times 9) + 1 = 8.3$.

Table 3: Enemy targets, which are the same assets as defenders, geographical coordinates and the value of targets (Brun et al., 2005).

Target	Longitude	Latitude	c	v	r	h	Value
Atsugi, JP	35°27'	139°27'	**4**	**7**	**6**	**5**	**7.7**
Misawa, JP	40°42'	141°25'	**8**	**5**	**7**	**5**	**8.2**
Okinawa, JP	26°20'	127°47'	**7**	**7**	**8**	**3**	**8.1**
Sasebo, JP	33°09'	129°44'	**7**	**8**	**7**	**7**	**8.9**
Tokyo, JP	35°41'	140°00'	**4**	**9**	**4**	**7**	**7.9**
Yokosuka, JP	35°17'	139°40'	**8**	**8**	**7**	**7**	**9.1**
Chinhae, ROK	35°08'	128°41'	**7**	**7**	**7**	**8**	**8.9**
Inchon, ROK	37°29'	126°38'	**3**	**6**	**5**	**4**	**6.9**
Kunsan, ROK	35°54'	126°37'	**10**	**7**	**9**	**10**	**9.7**
Osan, AB, ROK	37°06'	127°02'	**10**	**8**	**9**	**10**	**9.9**
Pusan, ROK	35°06'	129°02'	**8**	**7**	**8**	**10**	**9.4**
Seoul, ROK	37°27'	126°57'	**4**	**8**	**5**	**9**	**8.3**

Defense platforms

The two Agis defense cruiser have 20 standard missile-2 (SM2) and 10 standard missile-3 (SM3), and one Agis destroyer has 20 standard missile-2 (SM2). Defender has two ground defense systems. One of the systems is Patriot, which consists of eight mobile launchers, which are armed with four PAC-3 missiles, two PAC-2 GEM missiles and one PAC-2 missile. Another ground defense is the Todd system, which includes a launcher with 10 THAAD missiles.

Interceptor board

Table 4 shows the maximum range of Japanese interceptor missiles in this scenario, which are used by defense platforms.

Table 4: Range of Japanese interceptor missiles (Brun et al., 2005).

Interceptor type missile	Maximum range (km)
THAAD	250
PAC-2	160
PAC-2GEM	160
PAC-3	70
SM2	120
SM3	1200

Possibility of neutralization

The probability of neutralizing enemy missiles (interceptor missiles (Pk_a)) is estimated between 0.7 and 0.99. In this paper, the probability of neutralizing the enemy missile is considered to be equal to 0.99, ie (Pk_a=0.99).

Defender position

Figure 4 shows a grid map of the region between North Korea (enemy) and Japan (defender). Each circle in North Korea and Japan represents a goal. Each rhombus in North Korea represents a ballistic missile launch site. Triangular-shaped sea defense platforms can be located anywhere in the network at sea. Square-based ground defense platforms can be located anywhere on the ground except in North Korea. According to Figure 4, the defense points are located in latitude and longitude networks, each of which is about 60 nautical degrees. 304 Geolocation The natural geography prevents certain classes from being assigned to certain network locations. Defense systems can also be considered enemy targets.

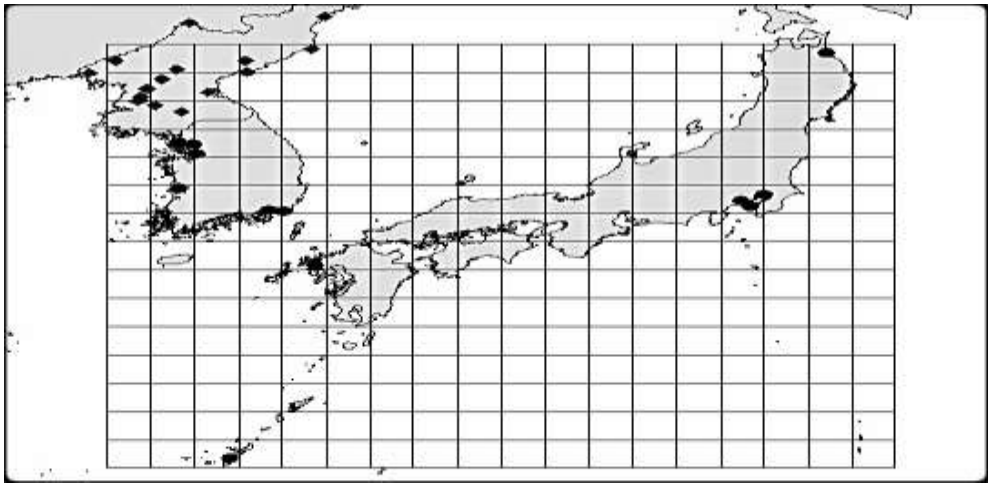

Figure 4: Network map of the region between North Korea and Japan as a network. Each circle in South Korea and Japan represents a goal. Each rhombus in North Korea represents a ballistic missile launch site (Brun et al., 2005).

Check the model in small dimensions:

In this section, the amount of damage mathematics in the state of attacker and defender concealment value is calculated with arbitrary data (with the help of external data et al. (2005)) by directly solving the MILP2 model in smaller dimensions. We have obtained the results of this section with Gomez software version (24.1.3) and a laptop with 8 GB of RAM and an Intel Core i5 4200U processor. Table (5) shows the number of defensive interceptor missiles in general, and Table (6) shows the location of the targets and missiles used by the attacker and defender, as well as the value of the targets using Equation (43) and Table (3). The value of the parameters is as follows:

$$k_a = 1, \quad (44)$$

$$p_{k_a} = 0.99, \quad (45)$$

$$shoot_p \leq 11 \quad , \quad shoot_p \geq 12. \quad (46)$$

By solving the MILP2 model, the values of the defender decision variables are obtained in one of the following steps:

$$X_{p_2,g_{97}} = 1, \quad (47)$$

$$X_{p_4,g_{92}} = 1, \quad (48)$$

$$R_{a_2,p_2,g_{97},d_3} = 1, \quad (49)$$

$$R_{a_1,p_4,g_{92},d_3} = 1, \quad (50)$$

$$Y_{a_1} = 1, \quad (51)$$

$$Y_{a_2} = 1. \quad (52)$$

This means that the defender must place his No. 2 platform in the 97th geographical location and intercept the second attack from this platform with the third defense option, and deploy platform number 4 in the 92nd geographical location and intercept the first attack from this platform with the third defense option, and the attacker will perform the first and second attacks, and the next steps will be obtained in the same way. For the defender in the $\overline{shoot_p} \leq 11$ mode, the value of the mathematical expectation of injury is 116.4 by solving the MILP2 model and 52.3 by solving the S2 model. Therefore, the value of the defender's concealment will be equal to 116.4 - 52.3 = 64.1, ie the defender will prevent damage by concealing 64.1. In the case of $\overline{shoot_p} \geq 12$, the value of the mathematical expectation of damage is

1.164 by solving the MILP2 model and 0.523 by solving the S2 model. Therefore, the value of the defender's concealment value is 1.164 - 0.523 = 0.641, which is one hundredth of the previous value. It is clear that in this case, too, the amount of damage is reduced by hiding operations. The difference between the two is that when we allow the defender to intercept and fire more missiles from each platform in each operation, we reduce the damage from the attacker to one percent.

Table 5: Interception platforms and number of interceptor missiles (Brun et al., 2005).

Defender platforms		Interceptor missiles					
Defensive platform class	Platform	THHAD	PAC-2	PAC-2GEM	PAC-3	SM2	SM3
Agis-C-G	CG47	-	-	-	-	20	10
Agis-C-G	CG48	-	-	-	-	20	10
Agis-D-D-G	DDG68	-	-	-	-	20	-
Patriot	Pbat1	-	8	16	32	-	-
THHAD	Tbat1	10	-	-	-	-	-

Table 6: Location of attack missiles, number of missiles, target areas and interceptor missiles in smaller dimensions (the value of targets is calculated from Equation (43) and Table (3)) (Brun et al., 2005).

Place of throwing the attacker	Attack type missile	Number of attack missiles	Target area	Goal value	Defender Interceptor Missile	Defender platform
Kanggamchan	No-Dong	10	Atsugi	7.7	SM3	CG48
Pyongyang	Scud-C	20	Okinawa	8.1	SM3	CG47
Kanggamchan	Scud-B	15	Osan	9.9	PAC-3	Pbat1
Kanggamchan	No-Dong	10	Tokyo	7.9	SM3	CG47
Chiha-ir	Scud-B	20	Inchon	6.9	PAC-3	Pbat1
Chiha-ir	No-Dong	10	Pusan	9.4	SM2-III	DDG68
Chiha-ir	Scud-C	15	Seoul	8.3	PAC-3	Pbat1

Conclusion

This paper is designed to optimize the defense of ballistic missiles against enemy attacks using mathematical programming models. In this article, the attacking forces (enemy) and the defender faced each other as two opponents in a two-player game. The attacker's plan is to inflict maximum damage and the defense plan is to minimize damage. This scenario creates a two-level linear programming model that has been transformed into a mixed integer linear programming solution. The more logical situation in the real world is for the defender and the attacker to be able to hide some of their information from each other, which in this article discusses the value of concealing and protecting information for both sides of the war. A war scenario includes launch sites, types of missile strikes, and a list of targets, and there are many limitations to hiding missile and missile strikes.

The sample studied in this article was related to the scenario of North Korea and Japan, the results of which were discussed. This design demonstrates the power of initiative and creativity to decide the missile defense scene and plan for a good defense. Existing defense planning is able to approximate and evaluate this scenario and must be implemented more accurately by existing optimization software. Joseph Caldwell Wylie says: "Definite planning for the future is the greatest and most heinous military error, and history bears witness to this claim" (Gary, 1999, p. 323). It cannot be planned in such a way that it is never surprised in the face of future developments; But with smart and forward-looking planning, the negative consequences of potentially surprising developments can be mitigated. In other words, it is possible to prevent catastrophic fractures and serious vulnerabilities in the face of future developments by establishing correct and principled defense capabilities (Gary, 1999, p. 333). This plan can be used for defense planning and estimating the value of information protection by further studies and experiments on different scenarios. It is hoped that this article will be effective in the development of military science and literature and will be considered and exploited by researchers and will be a step towards the advancement of applied mathematics in the field of defense.

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