



## Students' Errors in Solving Critical Thinking Problems on Linear Programing Topics

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### **Abstract**

Science and technology were growing more rapidly in the industrial revolution 4.0 era. Students must have some critical ability to deal with it. One of them is the ability to think critically. Critical thinking ability is the ability to use thinking skills to make reasonable decisions. Critical thinking skills can be developed through learning mathematics. This paper aims to identify and analyze students' error in solving problems that require critical thinking skills on the topic of linear programming. Data in this paper were collected using critical thinking skills tests and interview. The research method used is descriptive qualitative. The subjects in this paper were senior high school students in Palangkaraya. Four indicators of critical thinking skills used to analyze students' error. The results of the paper show the subject making several mistakes in solving problems. Some students make the same mistakes, some students can solve all problems even though the method used is not right, and the shape of the questions influences students' critical thinking ability.

**Keywords:** *Critical Thinking; Linear Programming; Mathematical Thinking*

### **Introduction**

The ability to think critically is an essential life skill (Galinsky, 2010). Critical thinking is one of the ten core life skills strategies and techniques listed by UNICEF, UNESCO and WHO (UNODC, n/d; World Health Organization, 1999). Therefore, the ability to think critically is crucial for students in facing the current era of the industrial revolution 4.0. Critical thinking is not only used in the academic field but is also very useful in everyday life.

Critical thinking is a process of reflective thinking that makes sense to focus on deciding something to believe or do (Ennis, 1993). Students who have excellent critical thinking skills are expected to be wise individuals in receiving and processing information so that they can be responsible for making decisions. Critical thinking is a habit and can be learned (Ruggiero, 2012: 21). Parents and teachers need to pay attention and help the students to develop their critical thinking. One that can be done to develop students' critical thinking skills is to provide opportunities for them to the discussion. When organized and managed well, discussions allow students "to develop critical thinking abilities and investigate questions that do not have simple answers" (Kauchak & Eggen, 1998, p. 250). "Critical thinking" as a form of higher-order thinking, as a form of problem solving, and as a part of the process of evaluating the

evidence collected in problem solving or the results produced by thinking creatively (Crowl et al., 1997; Lewis & Smith, 1993). So, critical thinking skills can illustrate other essential thinking skills.

In this paper, four indicators are used to measure students' critical thinking skills. The four indicators can identify assumptions, able to analyze relevant information or arguments, able to evaluate statements or answers, and able to determine the consequences of a decision. Students identify assumptions on a problem that are based on rational thinking and reliable information. Students analyze information or arguments that are relevant and not relevant to the problem given so that they can make decisions more effectively. Students evaluate a statement or answer to check the truth. Students consider every possible consequence of the decision to be taken. (Fisher, 2001; Siswono, 2018: 14; Norris & Ennis, 1985)

Critical thinking skills can be developed through a process of mathematics learning because mathematics has a structure and a strong and clear connectivity between its concepts to students who learn mathematics potential to be rational, logical thinking and critical (Firdaus et al., 2015). The ability to think critically in mathematics is called the mathematical critical thinking ability. In this paper, mathematical problems are used to assess students' critical thinking skills. The material used is linear programming. Linear Programming (LP) is a widely used mathematical technique designed to help managers in planning and decision making relative to resource allocations. It is a mathematical method for determining the way to achieve the best outcome in a given mathematical model for some list of requirements represented as linear relationships (Mahto, 2012). Linear programming and critical thinking have the same goal, which is to make decisions that provide the best results. Unlike other studies that examine the process and development of critical thinking skills, this study aims to identify student errors in solving critical thinking problems on linear programming material.

### ***Methodology***

The research method used is descriptive qualitative. The analysis referred to in this research is on students' critical thinking skills in solving linear programming problems. The subjects were senior high school students in Palangkaraya. They were students of 11th grade. The subjects in this study amounted to 6 subjects chosen by purposive sampling technique. The data needed in this study was collected by giving a test. The questions are about linear programming problem. The problems contain critical thinking indicators; they can identify assumptions, able to analyze relevant information or arguments, able to evaluate statements or answers, and able to determine the consequences of a decision.

### ***Results and Discussion***

In this paper, indicators of mathematical critical thinking used can identify assumptions, able to analyze relevant information or arguments, able to evaluate statements or answers, and able to determine the consequences of a decision. The six subjects are given four problems related to linear programming containing indicators of critical thinking. The problems are shown in Table 1.

**Table 1 Items Developed Based on Critical Thinking**

Item	Problem
1	Mr. Yudi wants to plant corn and chilli in his land. The land has a length of 40 m and a width of 20 m. In order to have a good harvest, between plants, is at least 1 m apart. Mr. Yudi wants to share 80 corn and 1 kg of chilli for his brother. One plant can produce an average of 4 corns and 0.1 kg of chilli. Mr. Yudi planned to plant 30 corn plants and 40 chilli plants. However, he is confused to buy how much fertilizer so that the fertilizer used was following the number of plants. One corn plant needs 300 grams of fertilizer, and one chilli plant needs 200 grams of fertilizer to grow properly. Mr. Andi suggested buying 16 kg of fertilizer, Mr. Yusep suggested buying 18 kg, while Mr. Daru suggested buying 19 kg of fertilizer. Who do you think is the most appropriate suggestion? Explain!
2	To be admitted to a University, prospective students must fulfill several criteria. Namely, the math scores multiplied by eight plus the Biology scores multiplied by three must be more than 240, the math scores multiplied by two not less than 180 minus the biological scores multiplied by nine, the math scores multiplied by six plus the biological scores multiplied by seven is not more than 420. Ayuna's scores are 50 on math tests and 10 at biological test. Is Ayuna accepted in the college? Explain!
3	A nutritionist suggests Mrs Yuni who intends to run a healthy life to consume at least 10 IU (International Unit) Vitamin B complex and at least 15 IU Vitamin D. One pack of a meal I contains 2 IU vitamin B complex and 1 IU vitamin D. In comparison, one pack of meal II contains 1 IU of vitamin B complex and 3 IU vitamin D. The price of a pack of a meal I is IDR 5,000, and one pack of meal II is IDR 10,000. In order to incur the lowest daily costs, Mrs Yuni consumed two packs of a meal I and three packs of meal II. is Mrs Yuni decision right? Give your reasons!
4	A bakery received an order to make 150 donuts and 120 chocolate breads. To make ten donuts needed 3 cups flour A and 2 cups flour B, while to make 10 chocolate breads needed 3 cups flour A and 1 cup flour B. For other ingredients, both types of cake require the same dose. The bakery has 9 kg of flour A and 4 kg of flour B. 1 kg of flour equals 10 cups. If you are the owner of a bakery, can you fulfill the requested order? If so, give your reasons! If not, what should you do?

From the four problems in Table 1, item 1 is a problem about identifying the assumptions given. Item 2 is about analyzing information or arguments that are relevant to the problem. Item 3 is about evaluating the answers given whether the answers are correct or not. Item 4 is about determining the consequences of a decision in fulfill consumer demand.

For the first problem, the researcher adds some information that is not used in answering the problems to determine the identity ability of the subject. The researchers give three assumptions to solve the problem. The task of the subjects is to identify each assumption and choose the most appropriate assumptions. Some of the subjects get errors in understanding the problem. It can be seen as in one of the answers to the following subject.

<p>Tidak ada diantara mereka yang memiliki saran yang paling tepat! Karena sekalipun saran diantara mereka semakin baik, pupuk yang dibeli tidak akan memenuhi rencana Pak Yudi. Menurut saya, lebih baik membeli pupuk dalam jumlah sedikit serta menurunkan sedikit ekspansi dalam rencana menanam itu. Setidaknya mulailah dari langkah terkecil, karena kalau mulai dari atas, bisa lebih merugikan kalau gagal atau tidak sesuai.</p> <p>Logika: 18 kg = 1800 gr, sementara 1 jagung 300 gr dan cabai 200 gr. Dari situ bisa diketahui bahwa ketiga saran itu tidak tepat.</p>	<p>maupun salah</p>
<p>None of them has the most appropriate advice. Because the fertilizer purchased will not fulfill Mr Yudi's plan. In my opinion, it's better to buy fertilizer in small amounts. At least start from the smallest step. If you start from a big step (planting in large numbers) it will be more detrimental if it fails. Logically 18 kg = 1800 grams, while 1 corn requires 300 grams of fertilizer and 1 chili requires 200 grams of fertilizer. From this information it is known that the three suggestions given are not appropriate.</p>	

Figure 1 Subject's answer for item 1

The subject cannot understand the given problem. It makes the subject unable to process the information provided. That inability makes the subject fail in identifying the assumptions that exist in the problem. The subject considers the three assumptions given are wrong. This shows that the subject does not fulfill the first indicator, which can identify the assumptions.

Furthermore, for the second problem, subjects were asked to analyze information about criteria to be accepted at a university. Based on these criteria, the subject determines whether a participant named Ayuna is accepted or not. Some students can answer correctly and some others cannot solve the problem. Here is one of the answers of students who cannot solve problems.

<p>Nini math = x -ll- bio = y <math>8x &gt; 240, 3y &gt; 240</math> <math>2x - 9y \geq 180</math> <math>6x + 7y \leq 420</math> Ayuna = 50 for math, 10 for bio <math>8 \times 50 = 400 \checkmark</math>   <math>2 \cdot 50 - 9 \cdot 10 = 10 \times</math> <math>3 \times 10 = 30 \times</math>   <math>6 \cdot 50 + 7 \cdot 10 = 370 \checkmark</math> Ayuna tidak diterima karena tidak memenuhi 2 kriteria (yang disilang)</p>	<p>Math score = x Biology score = y <math>8x &gt; 240, 3y &gt; 240</math> <math>2x - 9y \geq 180</math> <math>6x + 7y \leq 420</math> Ayuna: math score = 50, biology score = 10 <math>8 \times 50 = 400 \checkmark</math>   <math>2 \times 50 - 9 \times 10 = 10 \times</math> <math>3 \times 10 = 30 \times</math>   <math>6 \times 50 + 7 \times 10 = 370 \checkmark</math> Ayuna was not accepted because she did not fulfill 2 criteria (which are crossed)</p>
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Figure 2 Subject's answer for item 2

Based on the answers above, the subject cannot write information on the problem into mathematical model correctly. It can be seen from the subject's answers in the third and fourth lines. The correct mathematical model is  $8x + 3y > 240$  and  $2x > 180 - 9y$ . That error causes the subject's final answer to be wrong. The subject answered Ayuna was not accepted at the university. The correct answer is Ayuna accepted. The subject failed to analyze the information provided. So, the subject does not fulfill the second critical thinking indicator, which can analyze information or arguments.

For the third problem, some subjects answered correctly and some answered incorrectly and even did not answer at all. In the third problem, the subject is asked to evaluate the answer to a problem. Researchers give incorrect answers to see the subject's response. Below is one of the subject's answers.

9. Pakny Sedikit 10IU Vit B kompleks 15 IU Vitamin D  
 Satu sachet Makanan I = 2 IU Vit B kompleks dan 1 IU Vitamin D  
 II = 1 IU Vit B kompleks dan 3 IU Vit D  
 1 bungkus Makanan I = 5000  
 II = 10.000  
 Misalkan Ibu mengkonsumsi 2 bungkus Makanan I dan 3 bungkus Makanan  
 $\Rightarrow$  10.000 dengan 4 IU Vit B dan 1 IU Vit D; 30.000 dengan 3 IU Vit B dan 9 IU Vit D.

At least 10 IU of Vitamin B complex and 15 IU of Vitamin D  
 One sachet of meal I = 2 IU Vitamin B complex and 1 IU Vitamin D  
 One sachet of meal II = 1 IU Vitamin B complex and 3 IU Vitamin D  
 1 pack of meal I = IDR 5,000  
 1 pack of meal II = IDR 10,000  
 For example, Mrs. Yuni consumes 2 packs of meal I and 3 packs of meal II  
 $\Rightarrow$  IDR 10,000 for 4 IU Vitamin B and 1 IU Vitamin D;  
 IDR 30,000 for 3 IU Vitamin B and 9 IU Vitamin D.

Figure 3 Subject's answer for item 3

The subject has written information based on the problem correctly. However, after writing down the information, the subject did not give any answers. The subject did not determine whether the decision taken by Mrs Yuni was correct or not. It appears that the subject does not understand the purpose of the problem given. The answer shows the subject cannot process information and relate it to the problem to solve it. The subject failed in evaluating the decision taken by Mrs Yuni. So, the subject does not fulfill the third critical thinking indicator, which is able to evaluate statements or answers.

For the fourth problem, all subjects can answer the problem. However, the solution used by the subject is not following the linear program problem solving step. The steps in solving linear program problems are making a mathematical model, determining the objective function, making a graph, determining the optimum value of the objective function. Here is one answer from the subject.

1. Donut = x, chocolate bread = y, 1kg = 10 cup  
 tepung A = 9kg, tepung B = 4kg  
 $3x + 2y \leq 10$  ← Donut  
 $3x + y \leq 10$  ← chocolate bread  
 $x \leq 9kg \rightarrow x \leq 90cups$   
 $y \leq 9kg \rightarrow y \leq 90cups$   
 $3x + 2y = 10$        $3x + y = 10$   
 $x = 0$  (0,5)       $x = 0$  (0,10)  
 $y = 0$  (3,3,0)       $y = 0$  (3,3,0)

Answer: can't fulfill the order  
 flour A stock = 90 cups, flour B stock = 40 cups  
 150 Donuts = 45 cups flour A  
 30 cups flour B  
 120 chocolate bread = 36 cups flour A  
 12 cups flour B  
 What is needed to make 150 donuts and 120 chocolate breads are 81 cups of flour A and 42 cups of flour B. However, flour B is only 40 cups. So, in order to fulfill orders must buy 2 cups of flour B.

Figure 4 Subject's answer for item 4

Figure 4 in the red box is an answer from the subject who tried to use the linear programming's step in solving the problem. The subject presupposes donuts as  $x$  and chocolate bread as  $y$ . In the next step when making mathematical models, the subject use  $x$  as flour A and  $y$  as flour B. The subject is not consistent in making presuppositions. The subject could not complete the graph and determine the area of completion due to a previous error. Other than that, the subject did not understand how to apply the symbol of inequality in the Cartesian plane. Finally, students solve the problem using logic like the answer in the blue box. Students decide to buy 2 cups of flour B so that the order can be fulfilled. This shows that students still have difficulty in making mathematical models and drawing graphics. So, the subject fulfills the fourth indicator even though it is not by use of linear programming steps. The subject makes the right decision to fulfill the order.

Other subjects make different decisions even though both have the same answer that B flour is less than 2 cups as shown in figure 5.

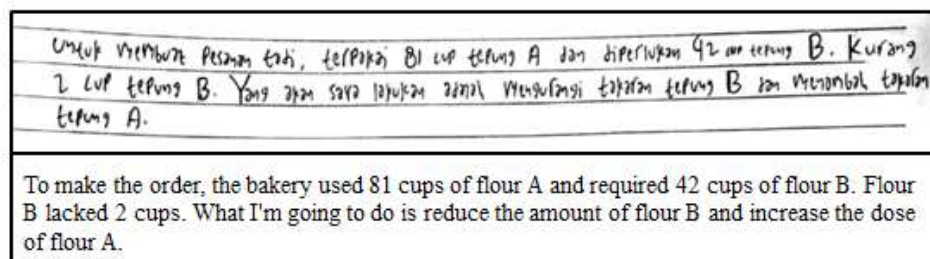


Figure 5 Other Subject's answer for item 4

The subject chose to reduce the dose of flour B in making cakes. The subject did not consider the consequences of the decision taken. If the flour dosage is changed, then the bakery cake recipe also changes. This can result in reduced quality of the cake produced. The decision taken by the second subject is not right. This shows that the second subject does not fulfill the critical thinking ability indicator, which is able to determine the consequences of a decision.

Researchers have examined all the subject's answers. Then, researchers summarize the results of the subject's answers and change it into percentages. Table 2 shows the results of the subject's answers based on indicators of critical thinking skills.

**Table 2 The Results of the Percentage of Critical Thinking Skills**

Item	Indicators of critical thinking skills	Percentage of subjects who fulfill the indicator	Percentage of subjects who do not fulfill the indicator
1.	Able to identify assumptions.	50%	50%
2.	Able to analyze relevant information or arguments.	83%	17%
3.	Able to evaluate statements or answers.	50%	50%
4.	Able to determine the consequences of a decision.	50%	50%

Based on Table 2, most subjects have difficulty in solving linear program problems that contain indicators of critical thinking skills. The indicator with the highest percentage that is fulfilled by the subject is the second indicator, which is able to analyze relevant information or arguments. For the first, third and fourth indicators, only 50% of the subjects who fulfill it. This shows that the subject is still

tricky in identifying assumptions, evaluating statements or answers and determining the consequences of a decision. The subject's limitations influence the inability of the subject in understanding information or problems. Some subjects can solve the problem to the point of being able to write down all the information in the problem correctly. However, the subject has difficulty at the crucial stage of solving the problem according to the problem.

These results show that the subject's critical thinking skills are still low. Critical thinking skills are essential to developing. This is consistent with research from Hove (2011) that explicit instruction in, and practice of, critical thinking strategies in the high school classroom can improve student academic performance. Adoption of critical thinking strategies can also prepare students for the rigours of university life, as well as helping them develop the skills necessary to compete economically in a global environment (Taylor, 2012).

### **Conclusion**

Based on the results and discussions, it can be concluded that the critical thinking skill of student in solving the problem for the topic of linear programming is still low. The number of students who able to analyze relevant information or arguments is 83%, and only 50% of subjects able to identify assumptions, able to evaluate statements or answers, and able to determine the consequences of a decision. According to this result, the indicator that is most fulfilled by the subject is the second indicator, which is able to analyze information and arguments. Meanwhile, the remaining indicators are only filled with 50% of the subjects. This shows there are still many subjects who have low critical thinking skills. To develop critical thinking skills can be done by more often giving non-routine problems that contain critical thinking skills. If students are accustomed to thinking critically, then when there are problems, students will solve it easier and faster.

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